

Learning and Teaching in Mathematics Education 4.0

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3D Animation

Not only a rat can punch holes

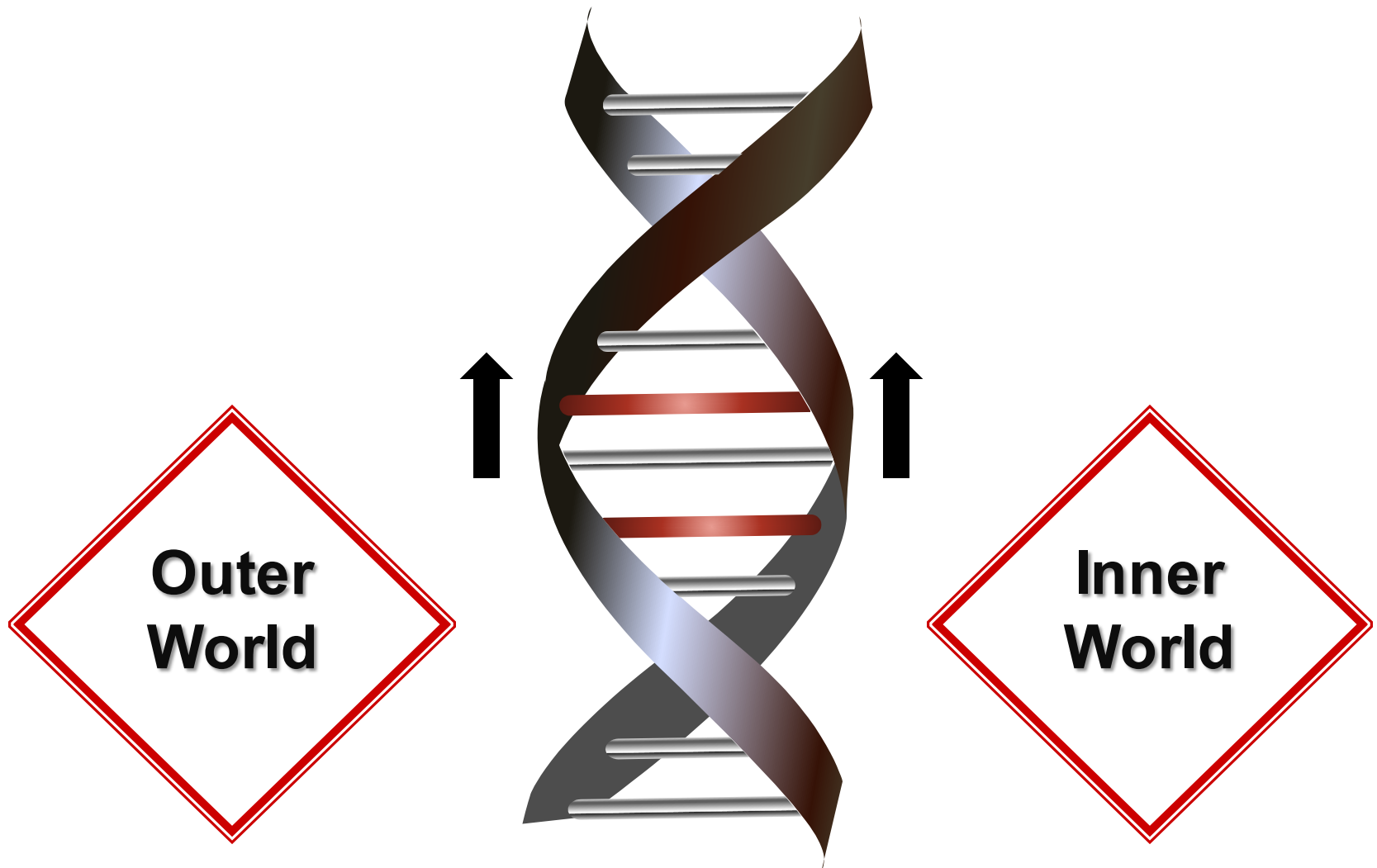
Coordinate battleship

Content

- I. Mathematics Competency as the Goal of Learning in Math. Ed. 4.0
- II. The principles of Designing Teaching for Competencies :
The Case of MGA in class
- III. The Research and practice of Just Do Math Program in Taiwan
- IV. Remark

I. Mathematics Competency as the Goal of Learning in Math. Ed. 4.0

Math Education 4.0



Outer World Issues

Industry 4.0 (AI, Big Data, Globalization)

- Math in Industry
- Math in Workplaces
- Math in Daily Life
- Working Style & Problem Solving

cf. (Gravemeijer.K & Stephan.M & Julie.C & Lin,F.L & Ohtani.M ,2017) What Mathematics Education May Prepare Students for the Society of the Future?

Inner World Issues

Societal agreement on goals of learning mathematics

i.e. Agreement on the perspectives of what is :

- Math ; for all
- Math learning
- Good math teaching
- Efficient teacher preparation
- Effective friendly learning environment for TPD
- National Assessment

Characterizing Math Ed 4.0

Characterization of Mathematics Education 4.0 depends on the **societal agreement on the goals of learning mathematics** that meets the demanding of the outer and inner issues .

e.g. A crucial issue in
Math. ED. 4.0

**What mathematics do we
teach when computer do
all calculation ?**

An alternative question

What is involved in doing mathematics in the real world ?

The four steps (Wolfram,2010)

1. Recognizing where mathematics applicable
2. Translating practical problems into mathematical problems
3. Solving mathematical problems.
4. Interpreting and evaluating the outcomes

Computer can do #3 mainly ;

Computer can't do, #1, #4.

A Decade-wise Inclusive Evolution

- Teaching for Understanding ,70'
- Teaching for Problem Solving ,80'
- Teaching for Modelling ,90'
- Teaching for Competencies ,00'

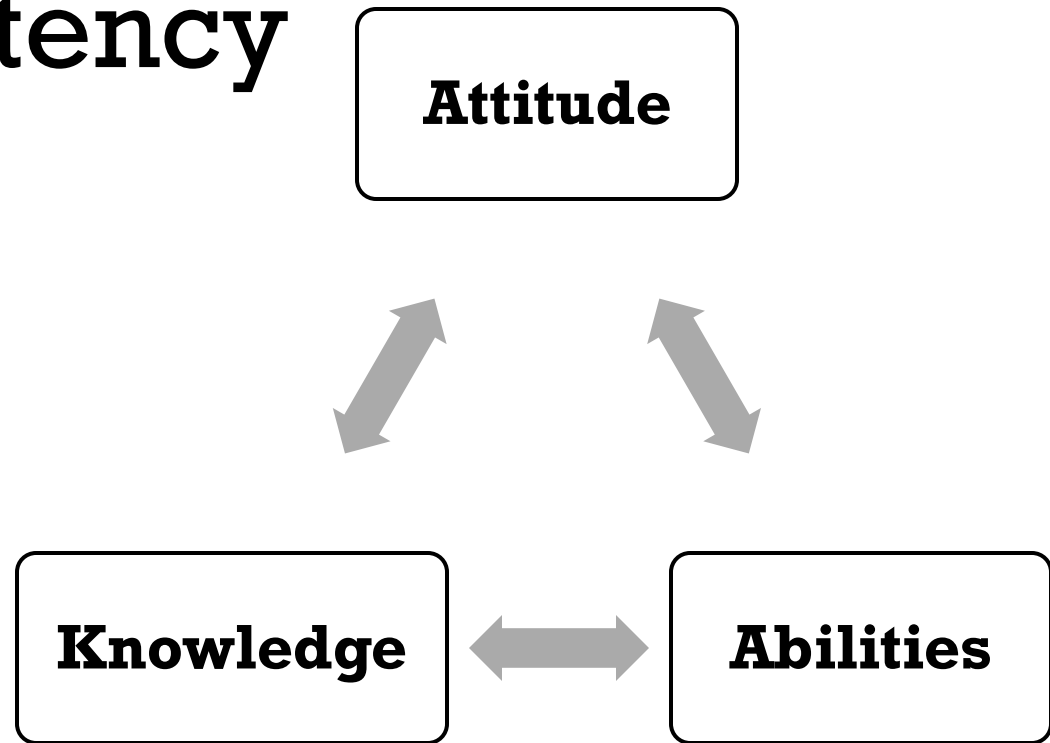
Key Focus

“Mathematics Competency” as the key word of societal agreement on the goals of learning mathematics.

What is Mathematics Competency?

In general

Competency



Cf. MOE(2019), National Curriculum 2019, MOE, Taiwan

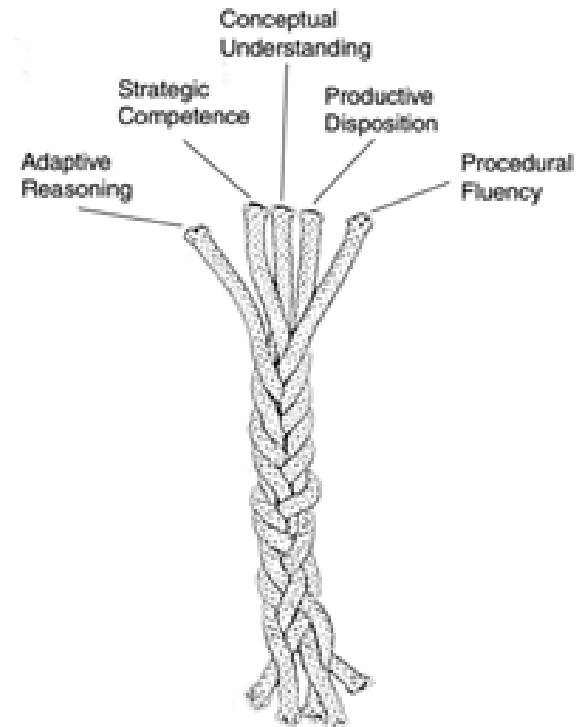
Mathematics Competencies

≈ Mathematics Literacy

in PISA's Framework

Mathematics Competencies

- *Conceptual Understanding*
- *Procedural Fluency*
- *Strategic Competence*
- *Adaptive Reasoning*
- *Productive Disposition*



- Kilpatrick, J., Swafford, J. & Findell, B. (Eds). (2001). *Adding it up: Helping children learn mathematics*. Washington: National Academy Press. Pp 115-155

Mathematics Literacy

Interacting Aspects of Mathematics Literacy – Cognitive Symbiosis

Fundamental Sense

- Cognitive and Metacognitive Abilities
- Mathematical Thinking and Quantitative Reasoning
- Habits of Mind
- Language of Mathematics (including proofs as arguments)
- Information Communication Technologies (ICT)

Derived Sense

- Understanding the Big Ideas, Strands, and Substrands of Mathematics
- Nature of Mathematics
- Knowledge about Problem Solving
- Real-world Problems

LEADING TO FULLER PARTICIPATION IN PUBLIC DEBATE ABOUT SOCIO SCIENTIFIC OR SCIENCE, TECHNOLOGY, SOCIETY, AND ENVIRONMENT (STSE) ISSUES LEADING TO INFORMED DECISION AND SUSTAINABLE ACTIONS ON QUANTITATIVE PROBLEMS.

Cf. (Yore, Pimm, & Tuan, 2007). IJSME

A Metaphor

Mathematics Competency

One mathematically

sees the world

&

interact with the world

The Metaphor VS.

Wolfram's (2010) Four Steps

- Mathematically sees the world

vs.

#1 Can recognize where math applicable

#4 Having plausible sense about the outcome

- Mathematically interact with the world

Approaches; #2, #3

II. The principles of Designing Teaching for Competencies : The Case of MGA in class

How can Mathematic competencies develop ?

(1) Engagement

The first priority

- Attitude
- Confidence

(2) Making sense of the world(see the world)

Starter : the fundamental sense of math objects via

- Body Sense
- Linguistic Sense
- Mind Sense

(3) Interact with the world :

Approaches

- Simple thinking model
 - Inquiry
- } Active thinking

Making sense of the world

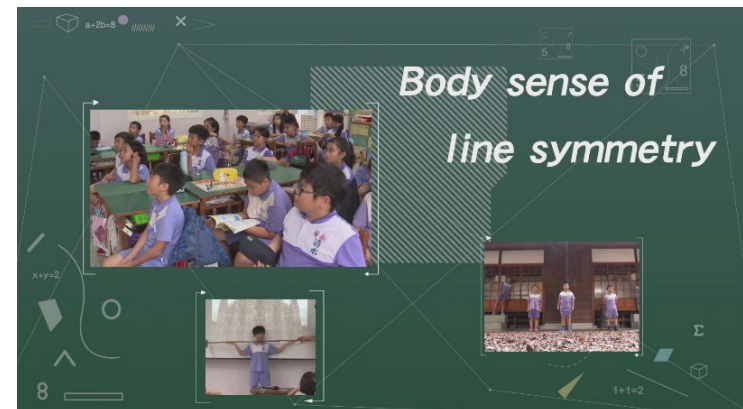
- 「Body」 sense

Sense of Weight :



Super Weighing : Developing sense of weight

Sense of Symmetry :



Body sense of Line Symmetry

Comment on the videos :

- Comparison activity in Videos
- Behavior : Perceptual Sense
- Awareness : Visual ↔ Analytic
- Affection : Relaxing

Making sense of the world

- 「Linguistic」 Sense

‘Language’ and ‘Number sense’

(Rectangular Number)



Comments on the video :

1. Classification Activity in the Video
2. Behavior : enactive, iconic, checking 9×9 table
3. Awareness : phenomena, language, embodied
4. Affection : corresponding mood with number
5. A good example of “seeing the world mathematically”

6. **Structural Analogous :**

Shapes : {point , line , rectangle}



Numbers : {1 , prime number , Composite number}

- 「Mind」 Sense

ex :

The diagonals of special quadrilaterals



Comment on the video :

1. Classifying Objects
2. Establishing Relations
3. Conjecturing Property
4. Developing deductive reasoning, involves the gradual evolution of enaction , referencing, description, condition refining , and precisising

Comment on the video :

5. Knowledge is co-construction
based on social interaction

6. What is math ?

**{Objects , Relations , Properties} \cup
{Thinking Model}**

More Videos of MGA in class

Catching the light and Shadow
(Similar Figures)

Rummikub of Mathematics

Identifying the Perspectives of Math. Ed. in the Videos

- What is mathematics?
- Perspectives of mathematics learning
- Perspectives of a 'good' mathematics teaching
- Perspectives of a 'good' teacher preparation
- TPD ?
- Assessment ?
- Curriculum ?

What is mathematics ?

For all

- Math is a language.
- Math is structured as
 $\{\text{Objects , Relations , Properties}\} \cup \{\text{Thinking Model}\}$
- Math is a science of patterns
- Math is within cultural activity
 - ➔ Comparison Activity
 - ➔ Classification Activity

Perspective of mathematics learning

1. Engagement

Cognition 90% ↑

Affection 90% ↑

Cf. (Lin, F. L., Wang, T. Y., & Yang, K. L., 2018). Description and Evaluation of a Large-Scale Project to Facilitate Student Engagement in Learning Mathematics. Studies in Educational Evaluation. Advanced online publication.

- Positive Attitude, Confidence
- the first priority of reforming Taiwan Math. Ed. :
Developing students' Intrinsic Motives toward Math

Perspective of mathematics learning

2. Continuous sense-making process
3. Co-construction Process
4. Conjecturing, Active Thinking Process

Perspectives of a 'good' mathematics teaching

- Providing learning activities for
 1. fully engagement
 2. sense-making continuously
 3. co-construction (S-S & T-S)
 4. promoting active thinking
e.g. conjecturing, modelling, inquiring

And

5. Diagnostic teaching, a necessary principle

Principles of Designing MGA in class

Integrating of

1~5

is a perspective of 'good' teaching.

which is the principles of “MGA in class”

III. The Research and practice of Just Do Math Program in Taiwan

The Just Do Math(JDM) program

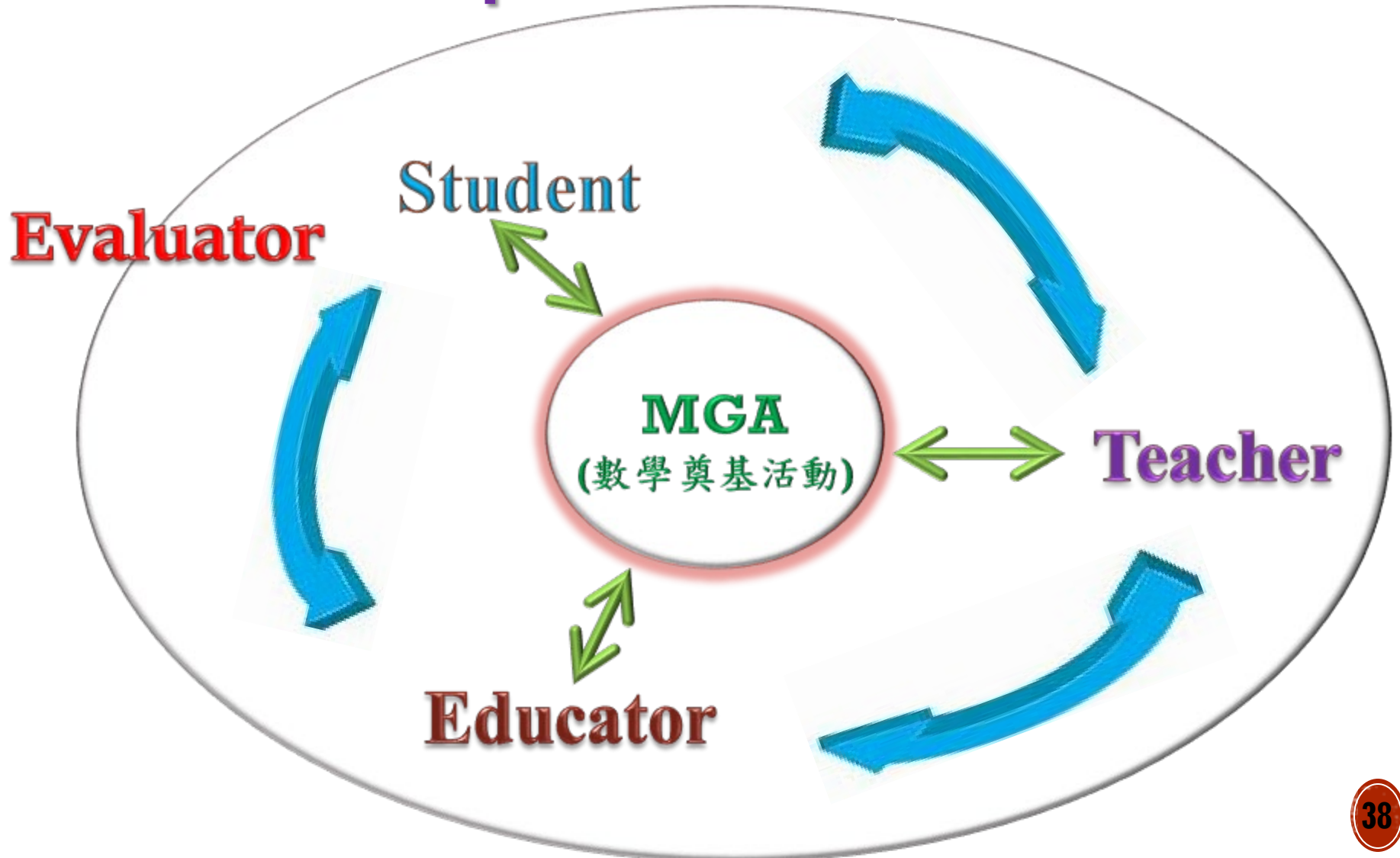
The Rationale and Principles of Designing Learning Activity

- Mathematics Grounding Activity (MGA)
& MGA in class (2014~)
- The 2nd phase of the Just Do Math (JDM)
program (2017~)

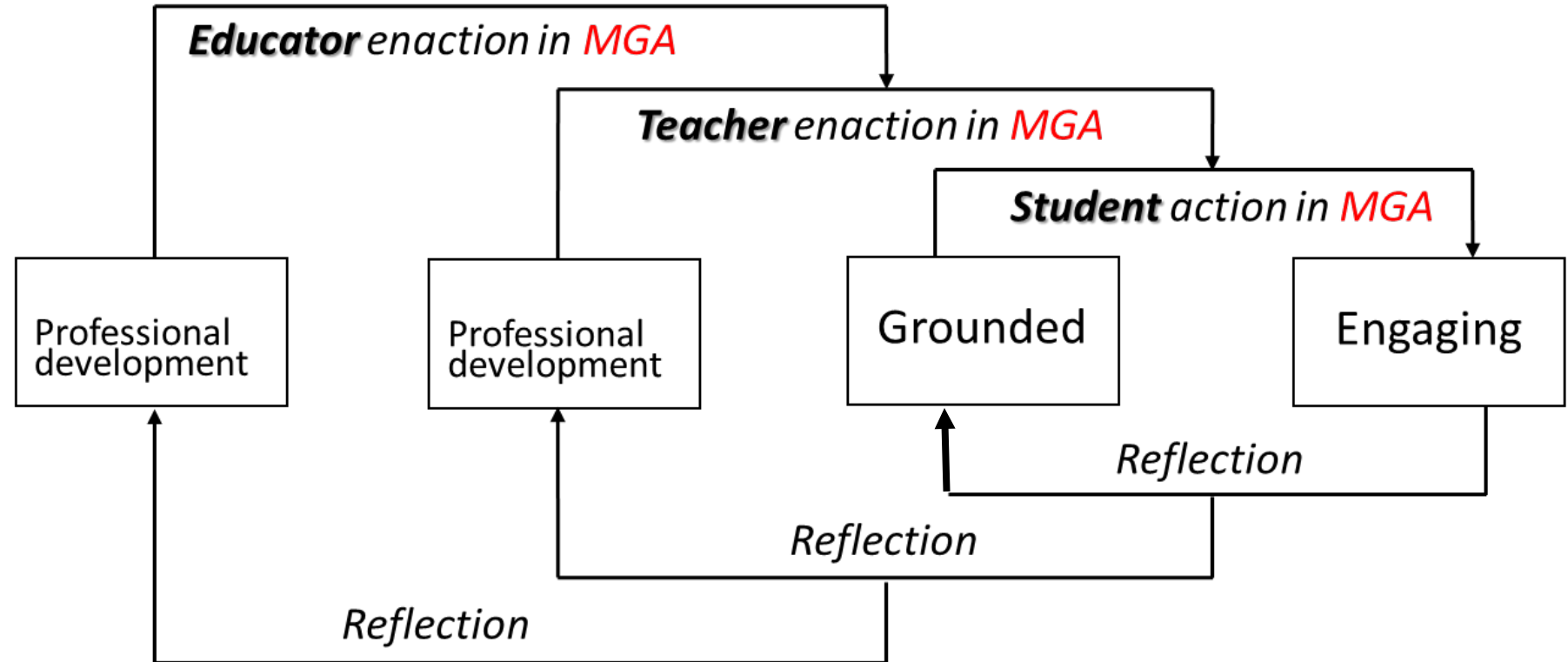
The 1st phase of JDM(2014~2017)

- Developing a set of MGA (175 modules)
for primary and junior high school math.
- Workshop for
 - MGA-teacher (N>13,000)
 - MGA-instructor (facilitator) (N>450)
 - MGA-designer (N>120 per year)
- Fun math. Camp.
 - Students (N>160,000)

Education Environment in 1st phase of JDM



Learning Environment in MGA



The Trans-learning Model in MGA.

Research Issues in JDM

Target of study

- Students
- Teachers
- Educator/Facilitator
- Educator-Researcher
- Curriculum

Specific Research Issues in

the 2nd phase of JDM(2017~)

1. The Workshops on 'Designing MGA-in class' :
3 x 2 Workshops in one year.
 - Each workshop with five sessions in one semester be hold in three regions .
 - Participant in each workshop :
15~30 MGA-teacher, instructors, facilitators

2. The workshop on co-lesson planning with 'MGA in class' as Reference (2018~)

- Study the efficiency of implementing the 'MGA in class'
- Study the facilitators' PD

3. Evaluation of the JDM program

- Internal
- External

IV. Remark

Lesson : MGA in class

One fully engages

in continuous sense-making

active thinking

and co-construct knowledge with others

Whenever misconception arise , be diagnosed.

Meaningful Learning

“ line point number , 線點數.”

“ Regular rectangle number , 正長數”

“ 1 only a point , all others can join a line”

— students’ co-construction in the video

“Rectangular Number”

One sees the world autonomously &
mathematically

Where Mathematic Applicable

“Everywhere in Campus can take line symmetric pictures”

— student’s reflection in the video

“Body sense of Line Symmetry”

Recognizing where mathematics applicable naturally !

MGA as LOGO

MGA is a Taiwanese Logo in Math. Ed.

- **JDM is an infrastructure for research and practice towards Math. Ed. 4.0.**

MGA as LOGO

- Functions of MGA in class , particularly the videos of MGA in class, can facilitate teacher preparation / TPD and promote “good” teaching in classrooms.
- Students are expected to make good sense of the world and actively interact with the world.

MGA as LOGO

- The 1st phase of JDM is a necessary process

The products ◇ 175 MGA

◇ 13,000 MGA-teacher

◇ Validating of MGA with 160K students

become the most powerful resources for working
in Math. Ed. 4.0.

變化中的數學課

just enjoy math

<https://www.youtube.com/watch?v=nTaG3iEX0JU>

Publications

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Thank You

