



A Holistic Model of Learning with Mathematics Grounding Activity

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Mathematics- Grounding Activities

A system of Generic Learning Activity

Conceptual
representative

Conceptual
developmental

Easier way
to learn

Enjoyable

(Mason & Pimm, 1984; Cheng, 2000)

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

A brief narrative

Holistic psychology is a trend in Mathematics Education. It pays attention on both affective and cognitive performances during learning mathematics.

A program that aims to resolve the national problems in Taiwanese school mathematics learning, i.e. low interest, low confidence, don't feel mathematics useful, but having high average performances in TIMSS study, had developed a set of learning modules (175 modules), the Mathematics Grounding Activities (MGAs). MGAs had been characterized with four features: enjoyable, easy, conceptually representative, and developmental.

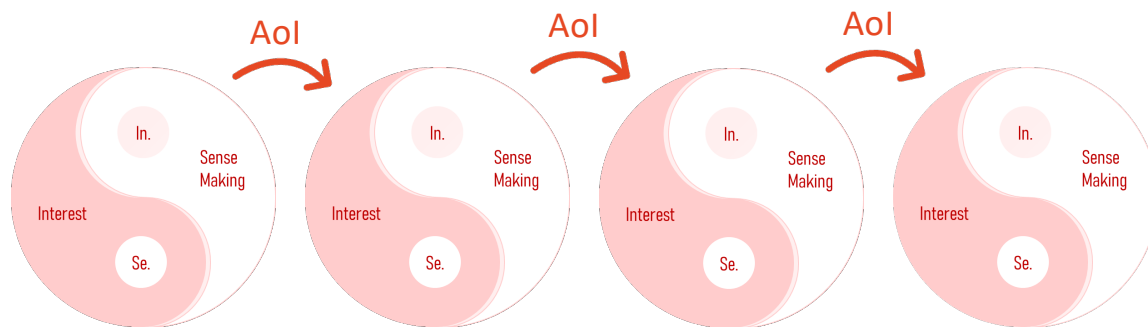
A brief narrative

The evidence for supporting the characterization were not only their designing principles but also empirical analysis based on their classroom teaching videos. Those MGAs-based teaching videos are supposed to be organized as a set of holistic teaching videos of primary and junior high school mathematics.

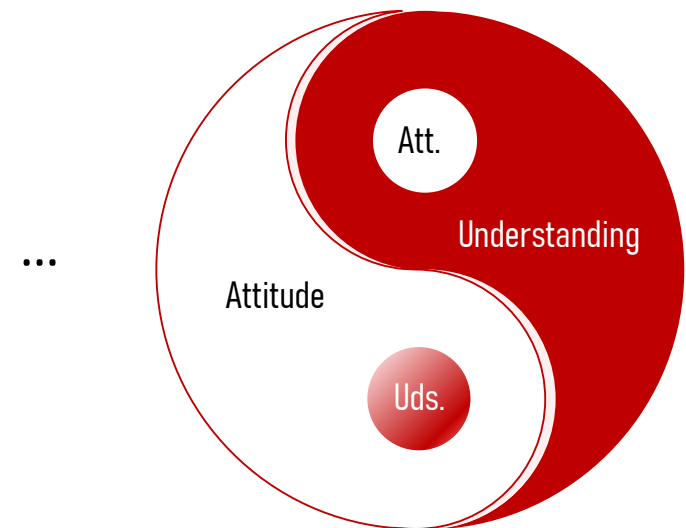
Based on a holistic teaching video: the rectangular number game, this lecture will present a holistic model of learning mathematics. The crucial issue is “how learners can be triggered and sustain their interests and meaningful learning during classroom lessons?” A key construct, the awareness of insufficiency has been identified and argued to be driving force triggering inter-evolution of affect and cognition in learning with MGAs.

■ A Holistic Model of Learning Mathematics ■

Awareness of insufficiency (Aol) drives the learning process



Awareness of insufficiency drives the learning process



MGA-in-class

Four principles of designing MGA-in-class :

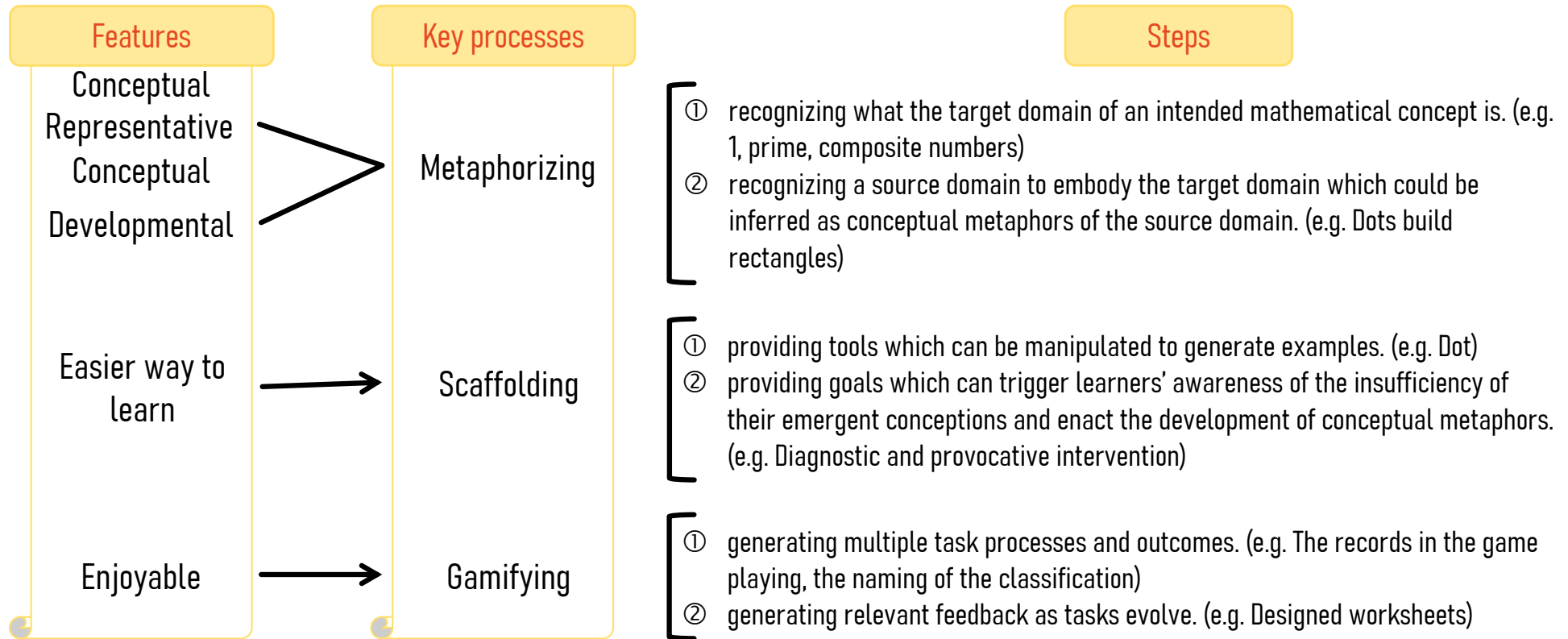
1. Triggering thinking principle: Triggering thinking for enhancing **intrinsic learning motivation** and developing positive learning attitude.
2. Sense making principle : Promoting students' **active** sense making **continuously**.
3. Co-constructing principle : Promoting discussing, thinking and co-constructing.
4. Diagnostic principle : Diagnosing conceptual understanding.

Learning with MGAs

Enactivist approach on learning with MGAs (Yang, Lin, & Tso, 2021)

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

Frame of designing MGAs



Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

Some Claims in Yang Lin, & Tso (2021)

1. “Learning is the systemic interactions between learners, tasks and social contexts, and learners’ motives integrated into the evolving tasks (Yang, Lin, & Tso, 2021).”
2. **Awareness of insufficiency** has been noticed in the learning process with MGAs (Yang, Lin, & Tso, 2021).

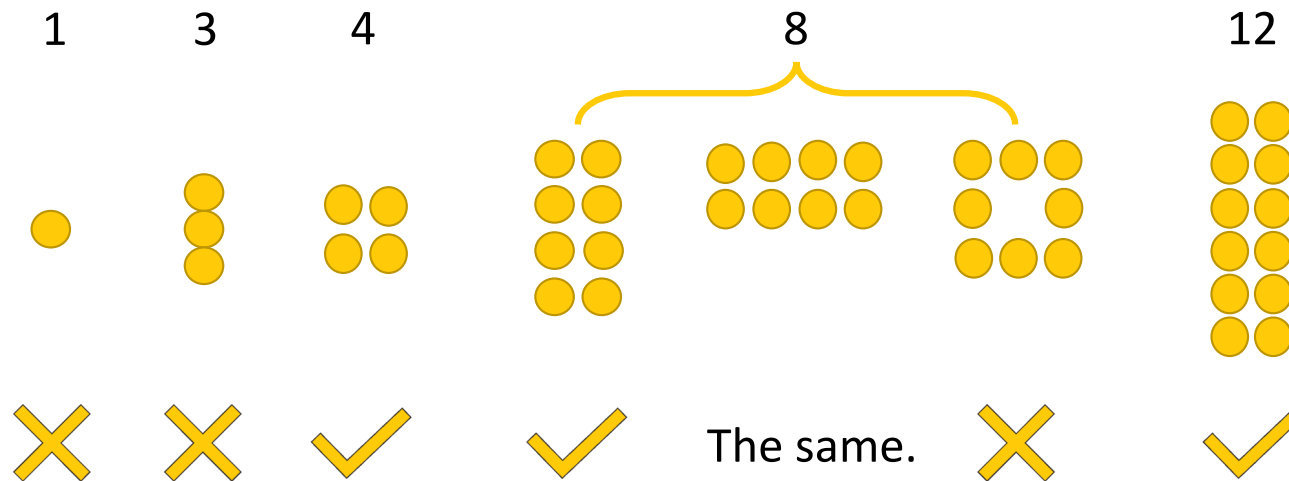
Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

Modelling the learning process with MGAs

Rectangular Number Game

The rule of the game

- In the game, we need to use the given number of the chesses to form a rectangle.



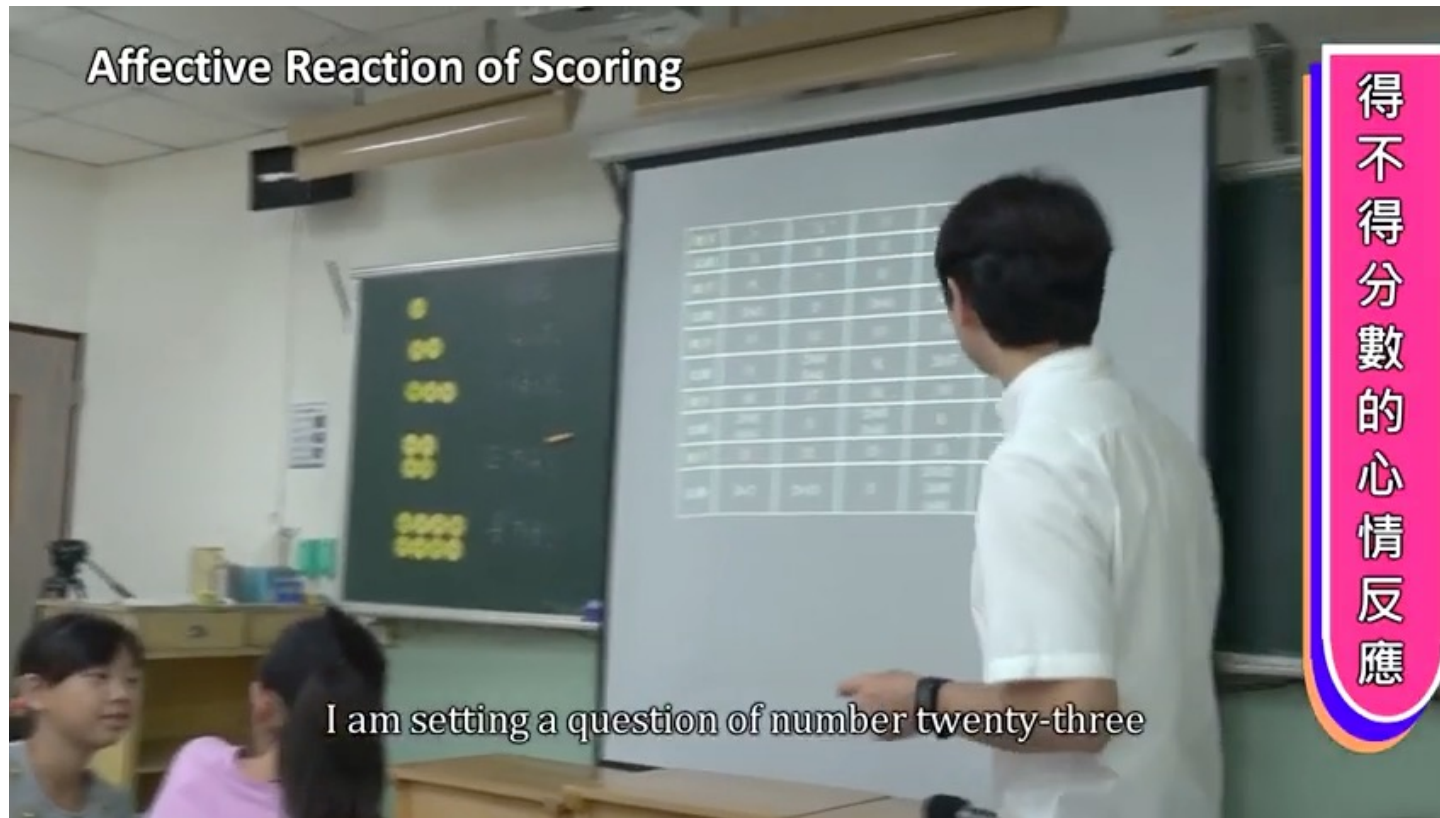
Rectangular Number Game

Worksheet



Video : Rectangle Number Game (RNG)

Insert RNG video: 08:46-10:55, 11:09-12:07



Linkage: <https://www.youtube.com/watch?v=f1miotpvqBo>

Modelling the learning process with MGAs

 **Methods: Insufficiency-based narrative analysis.**

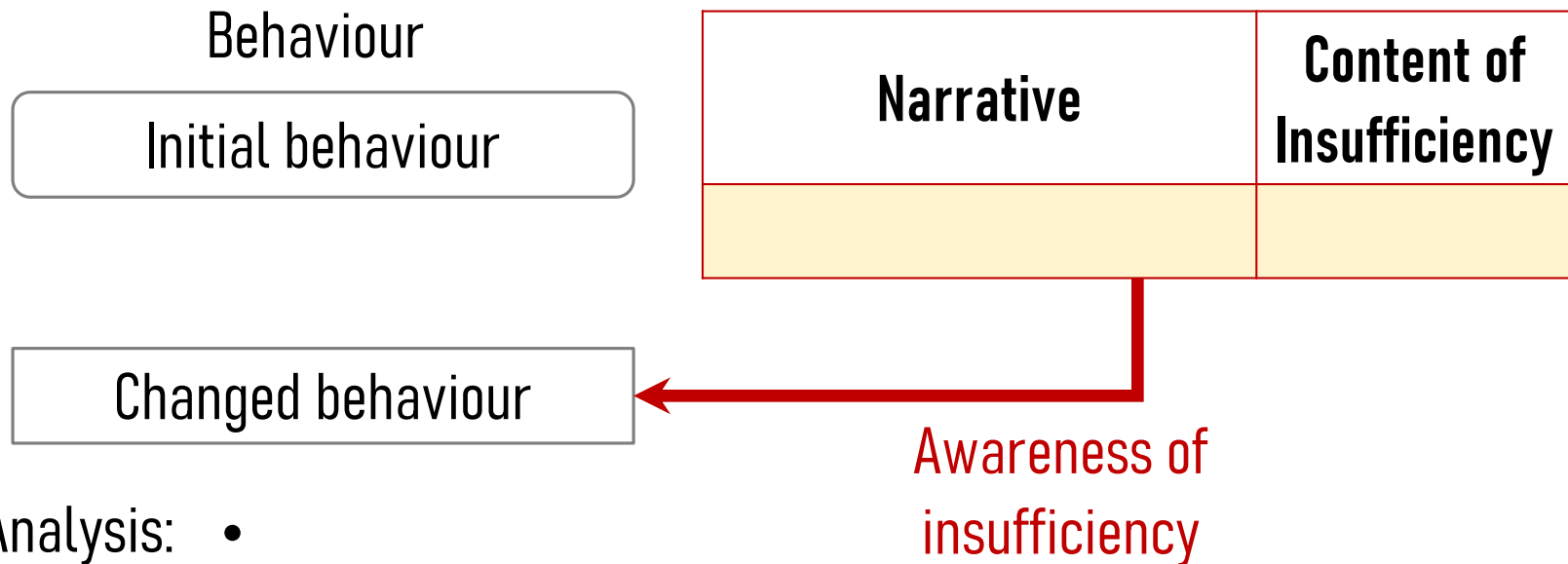
 **What is insufficiency-based narrative?**

- One insufficiency, one narrative
- Content of insufficiency: interpreted based on math thinking and process
- Types of awareness of insufficiency shown in the following table

Insufficiency	Self	Others
Awareness		
Autonomous	S-A	O-A
be Led	S-L	O-L

- Holistic analysis: inter-transition of emotion and cognition underline the changed behaviour

The insufficiency-based narrative analysis : A flow chart representation



- Holistic Analysis:
- -
 -

Representation of insufficiency-based narrative: a flow chart

Holistic (psychology) analysis

- Hannula's (2001) four construct
 - Meta-cognition, meta-emotion, cognitive emotion, emotional cognition
- Roth & Walshaw's (2019) Drama sense
 - Multi-perceptual representation

Summary of the lesson

Behaviour

Game rule practicing

Trial-and-error strategy

V-H vs. H-V perspectives

Different strategies used in the game play

Change to the division thinking



Content of Insufficiency

In cope with the rule.

Concept of perspectives

Demand enactive mode

Constrain by tools

Types of awareness of insufficiency

S-L

O-A

S-A

S-L

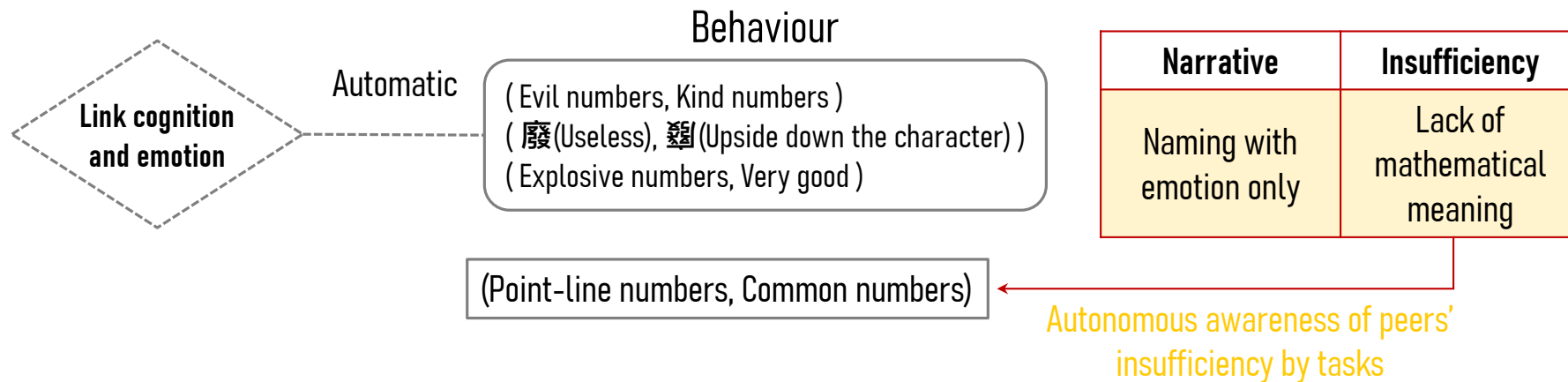
From the beginning of the game to the diagnostic intervention, interest and sense making interwoven as a unitary learning construct.

Summary of the lesson

Behaviour	Content of Insufficiency	Types of awareness of insufficiency
(Evil numbers, Kind numbers) (廢 (Useless), 顛 (Upside down the character)) (Explosive numbers, Very good) (thinking mathematically)	Lack of mathematical meaning	O-A
(Point-line numbers, Common numbers) (thinking mathematically)	Naming in complex	O-A
(Surprising numbers, Regular-rectangular numbers)	No scoring, so no motives to distinguish 1 from prime	O-L
1 vs. prime		

The whole class not only share the mathematics meaningful naming (Point-line numbers, Regular-rectangular numbers), but also has the potential to complete the nature number classification {1, prime, composite numbers}. 17

Insufficiency-based narrative analysis



Holistic analysis of 線點數 (Point-line numbers) :

1. The awareness of peers' insufficiency drive students' **eager** to name cognitively with geometric figure. (A brilliant performance)
2. The geometric feature of point-line numbers did not applied to the other category, it's a naming in complex (Vygotskian Thinking in complex).
3. Cognitive loading constrains the other category naming.

Insufficiency-based narrative analysis

Behaviour

(Point-line numbers, Common numbers)

Narrative	Insufficiency
Naming with different thoughts.	Naming in complex

(Surprising numbers, 正長數 (Regular-rectangular numbers))

Autonomous awareness of peers' insufficiency

Note:

正長數 (Regular-rectangular numbers)

正常數 (normal numbers)

Same Chinese pronunciation

Insufficiency-based narrative analysis

Holistic analysis on the socio-cultural of students' performance

Holistic analysis:

- The naming, point-line numbers, triggers the student's appreciation. In addition, the student is also aware of his peer's insufficiency. This gives him the opportunity to **meaningfully creates** the new terminology “正長數(Regular-rectangular numbers)”. The term 正長數(Regular-rectangular numbers) appears in the first time for all (**innovative**).
- Meaningful creation and innovations are well performance grounded with the coupled affect and cognition.
- His elaboration is to avoid his peers' misunderstood of 正長數(Regular-rectangular numbers) as 正常數normal numbers due to that the Chinese pronunciation of 正長(Regular-rectangular) is the same as 正常(normal), and 正常 is for normal. Language is a cultural element, and his elaborated attitude follows the norm of social interaction.

Insufficiency-based narrative analysis

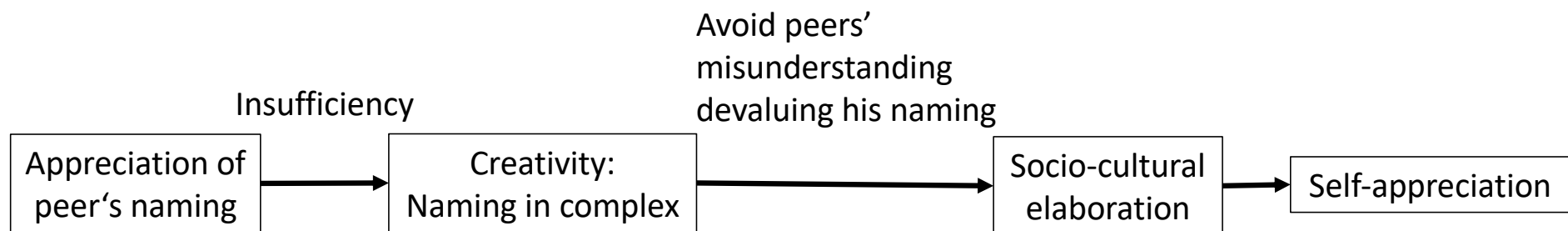
Socio-cultural analysis of the name 正長數 (Regular-rectangular numbers)

- In the moment while he elaborates his naming, his facial expression, voices and posture shows his high appreciation of his creative naming as the drama sense in Roth & Walshaw (2019). (Here shows that he is in the peak moment of learning).

Insufficiency-based narrative analysis

Socio-cultural analysis of the name 正長數 (Regular-rectangular numbers)

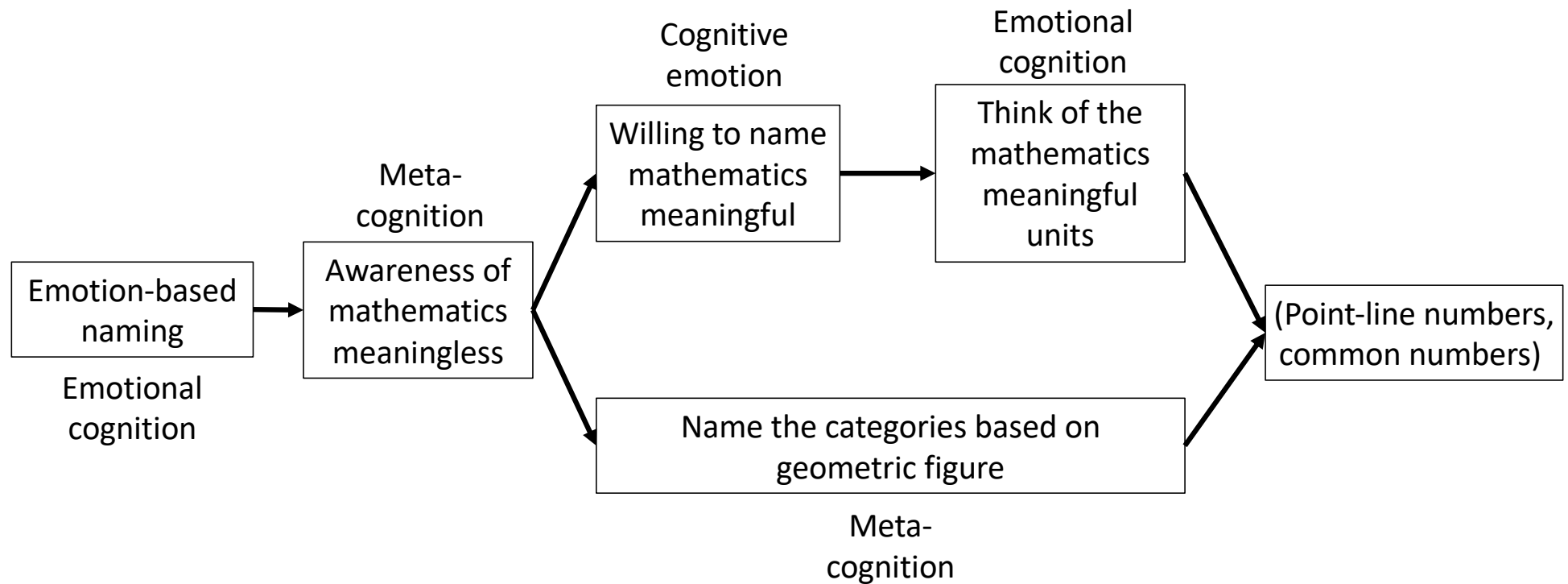
The learning process under socio-cultural context:



It is beyond learning in social context

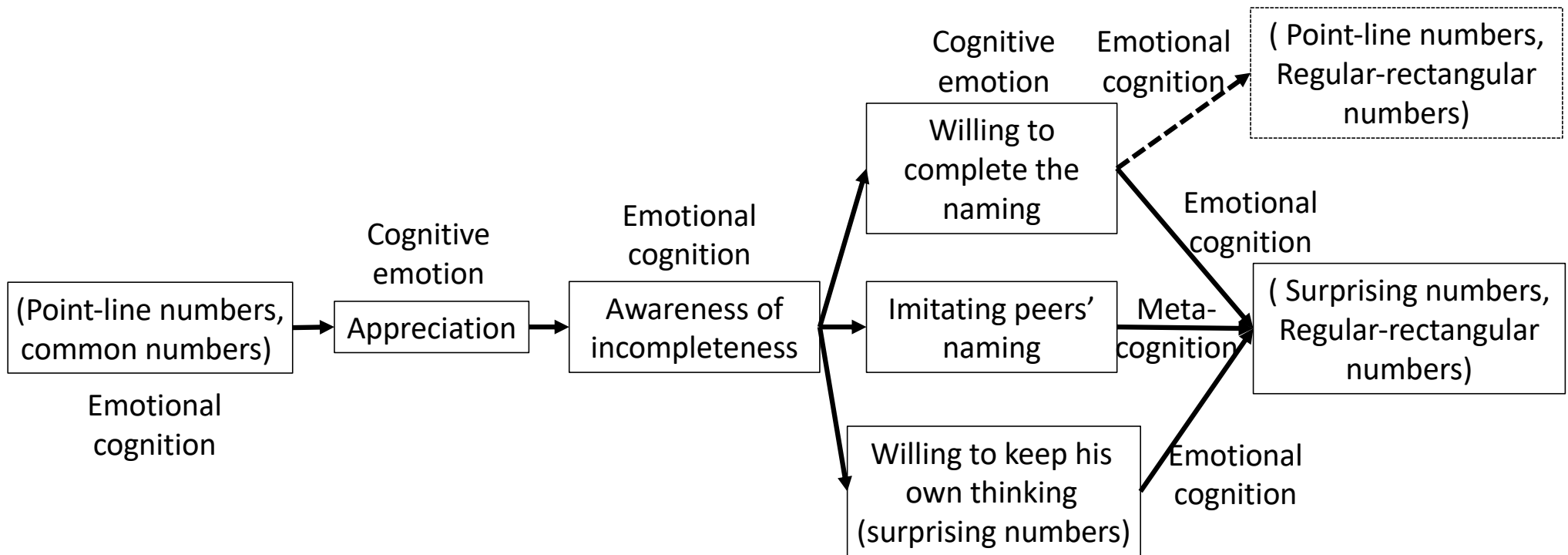
Insufficiency-based narrative analysis

The analysis referred to Hannula's (2001) four constructs



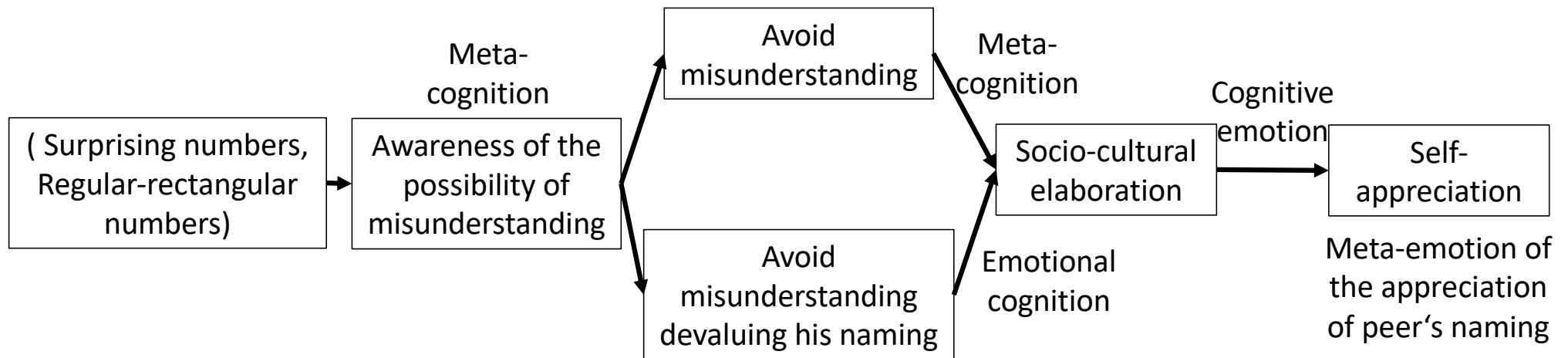
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Insufficiency-based narrative analysis

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Insufficiency-based narrative analysis

Behaviour

(Point-line numbers, Common numbers)

Narrative	Insufficiency
1 and prime are in the same category	No scoring, so no motives to distinguish 1 from prime

Point vs. line

Leaded awareness of game-based insufficiency by teacher intervention

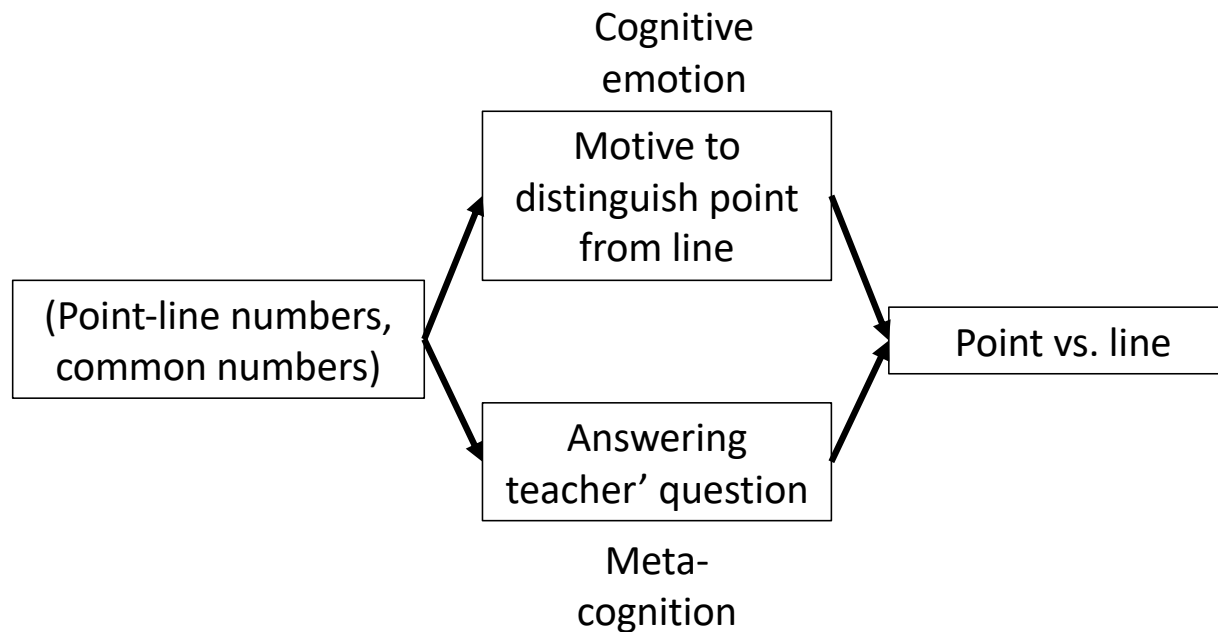
Insufficiency-based narrative analysis

Holistic analysis:

1. During the game, due to 1 and prime are no scoring numbers, so no motives for distinguishing.
2. After be leaded by teacher's provocative question, students pay attention to the collective knowledge, (point-line numbers, regular-rectangular numbers), and focus on the geometric figure to distinguish dot from line and to distinguish 1 from prime based on the different geometric objects.
3. When students can apply the collective knowledge to response to teacher's question and receive teacher's appreciative feedback, they enjoy the social interactive approach of learning.

Insufficiency-based narrative analysis

The analysis referred to Hannula's (2001) four constructs



Insufficiency-based narrative analysis

Discussion: The development of the scientific concept of prime numbers

1. What if the teacher asks the differences within the categories of regular-rectangular numbers? Maybe it will lead students to realize the inclusive relation of square and rectangle.
2. Based on the inclusive relation, students can really sense the isomorphism between {point, line, rectangle} and {1, prime, composite numbers}.
3. Next, we will lead students to describe their image of prime numbers.
4. Finally, we hope the description can develop students' scientific concept of prime numbers.

Results of the modelling

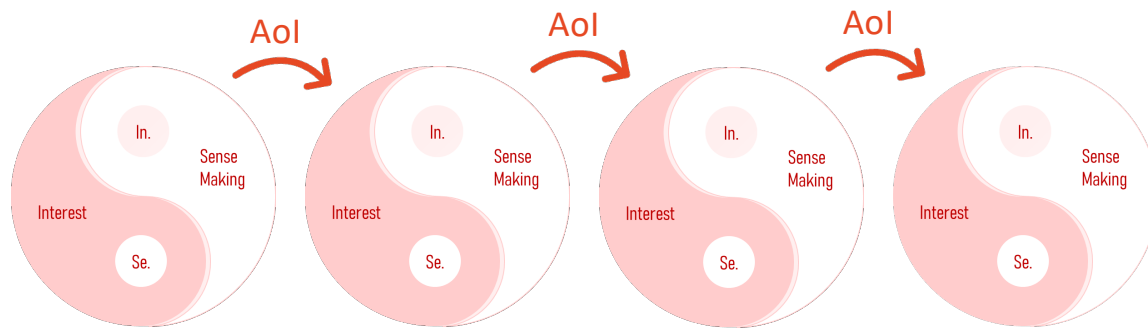
- The designing principles and the features of MGAs supports that students should make sense of the concept and the tasks should provide the opportunities to motivate students and trigger students' interest. Here, interest and sense making are interwoven as a holistic unitary learning construct, i.e. Interest * sense making, and serve as the starter of students' learning.
- Awareness of insufficiency drives the learning inter-evolved with affect and cognition. We can observe the positive developing inter-evolution of affect and cognition in the settings of MGAs, and the continuously positive development of the inter-evolution of affect and cognition steps forwards to the interwoven of productive disposition and understanding.

Results of the modelling

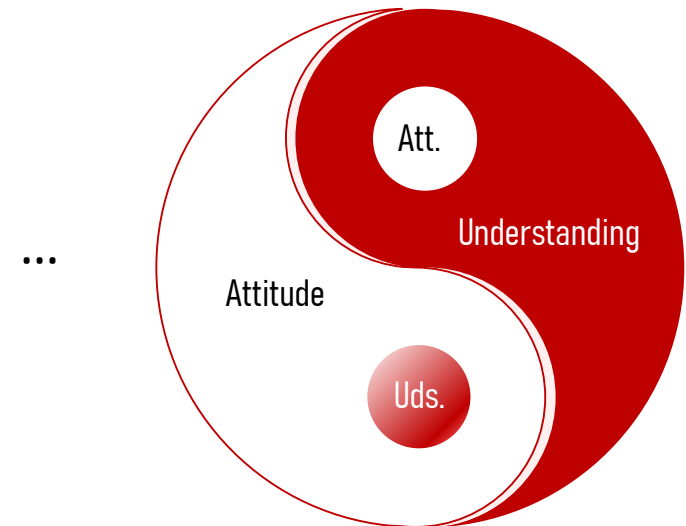
- Learning is a recursive self-integrative process. It starts from the holistic unitary of interest and sense making and then different kinds of awareness of insufficiencies will pave the way to nurture students' productive disposition and understanding. In the process of paving the way, the holistic unitary of interest and sense making recursively functions everywhere to develop the holistic unitary of productive disposition and understanding, and thus forms a recursive self-integrative process.
- Encouraging all students' self idea and providing representation opportunities are two strategies used in MGAs to sustain the inter-transition of affect and cognition.

■ A Holistic Model of Learning Mathematics ■

Awareness of insufficiency (Aol) drives the learning process



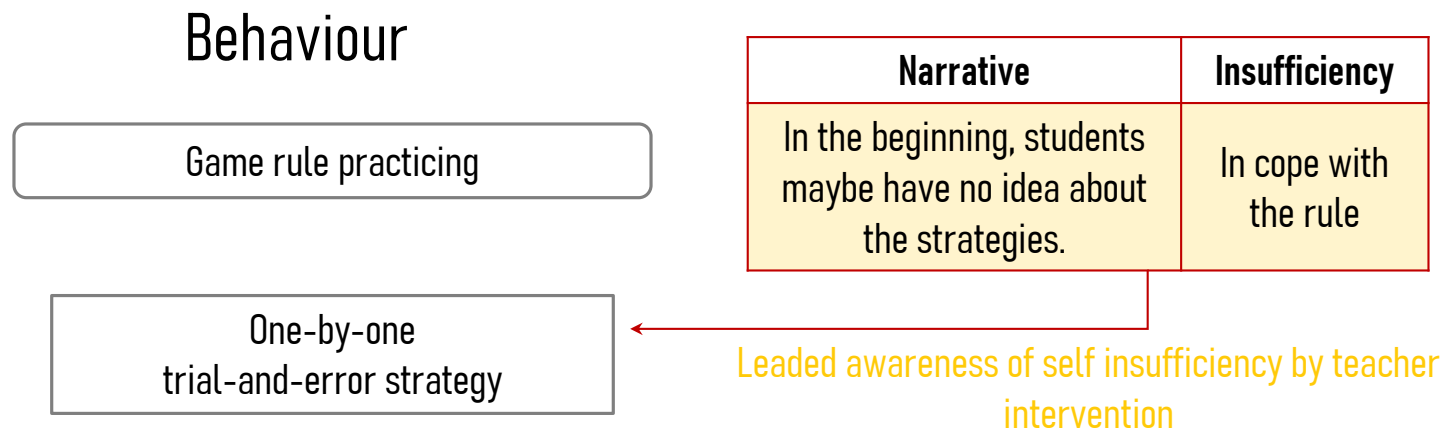
Awareness of insufficiency drives the learning process



Appendix

Insufficiency-based narrative analysis

Rule of the game

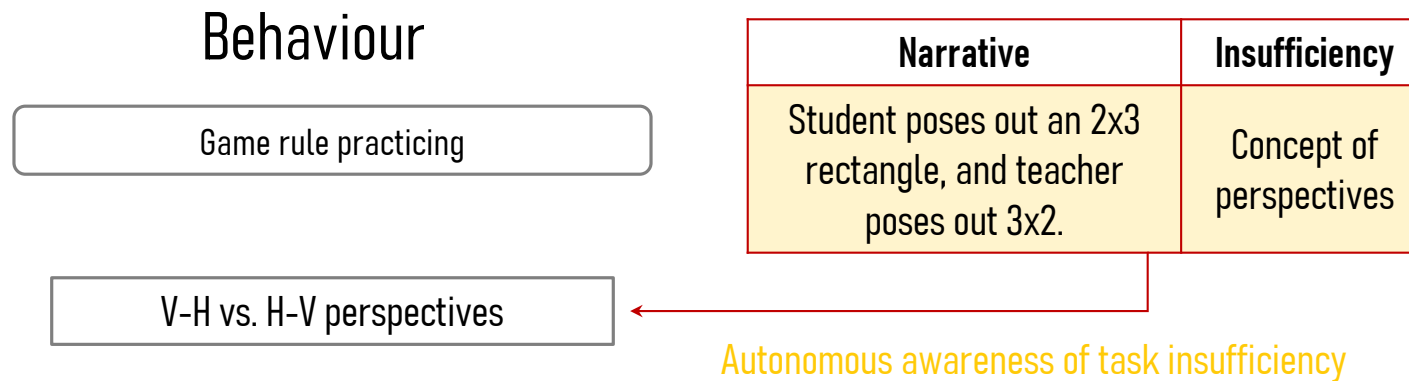


Holistic analysis :

The teacher performs the one-by-one trial-and-error strategy to the students in order to lead students feel hopeful, interest and make sense of the game.

Insufficiency-based narrative analysis

Rule of the game



Holistic analysis :

1. The rectangle posed by the teacher lets students feel the teacher's laziness because they feel that the teacher only rotates the student's result. He doesn't need to think. (This triggers students' body sense about the shape rotation)
2. This feeling triggers students to reject that 2x3 and 3x2 are different cognitively. (Let students make sense and build the rule)

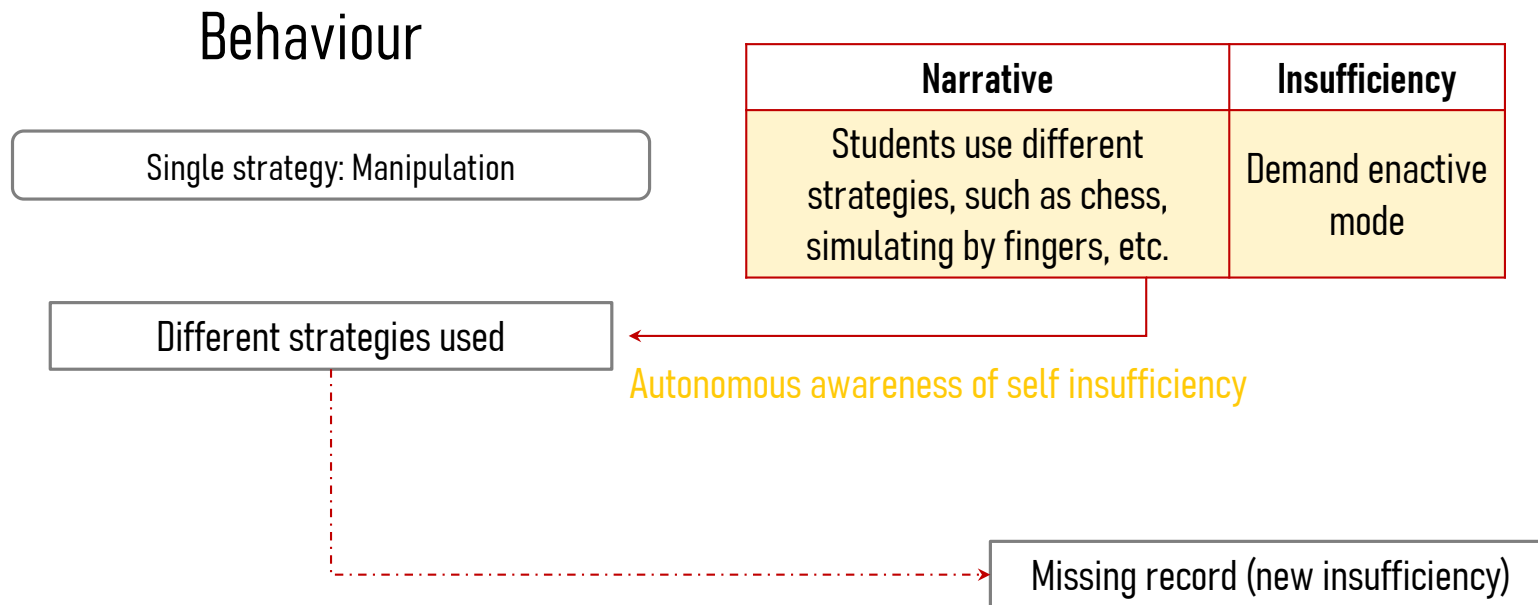
Insufficiency-based narrative analysis

Remark:

- From the usage of chesses, students can easily sense the exchange of multiplier and multiples, this is also an opportunities for students to realize the commutative law.
- However, if the teacher uses the cubic to replace the chesses, then the following phenomena may happen:
 - Students cannot shrink the cube into point as the chesses. This may confuse them and cannot lead them to treat 1 as point. However, in the situation, students can learn the concept of factors and divisors.
 - When students use cubes, then they will see that 2×6 and 6×2 are not the same. This will also change the rule of the game. However, this is also an opportunity for students to see that commutative law not always appropriate to use under different system.

Insufficiency-based narrative analysis

Game playing



Insufficiency-based narrative analysis

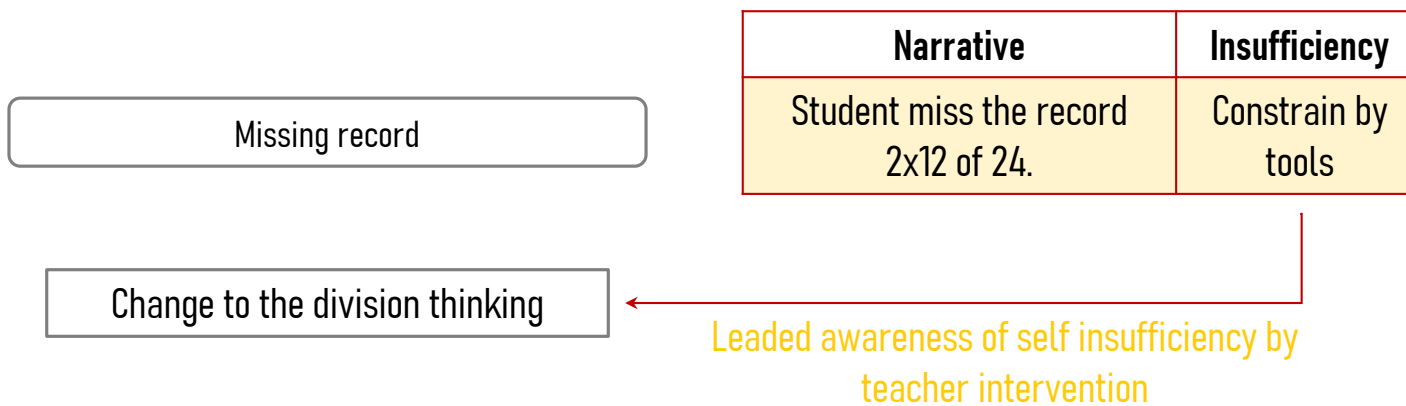
Game playing

Holistic analysis :

1. During the game play, the game triggers students' eager to win the game. The eagerness triggers students that they want to have fast or appropriate strategies for them to win. Students use different tools such as chesses, finger simulation (enact), drawing circle (iconic mode), multiplication table.
2. Students can switch tools and can keep playing the game, so they feel **easy and make sense of the game**, and the strategies can also lead students play the game fluently, which enjoys students.
3. In addition, during the game play, students will also use their peers' strategies, i.e. they will share the strategies with each other. **(The active co-construction between peers)**
4. However, new insufficiency will also happen, such as when students rely completely on 9x9 multiplication table, they might miss the multiplication beyond 9x9.

Insufficiency-based narrative analysis

Diagnostic intervention



Insufficiency-based narrative analysis

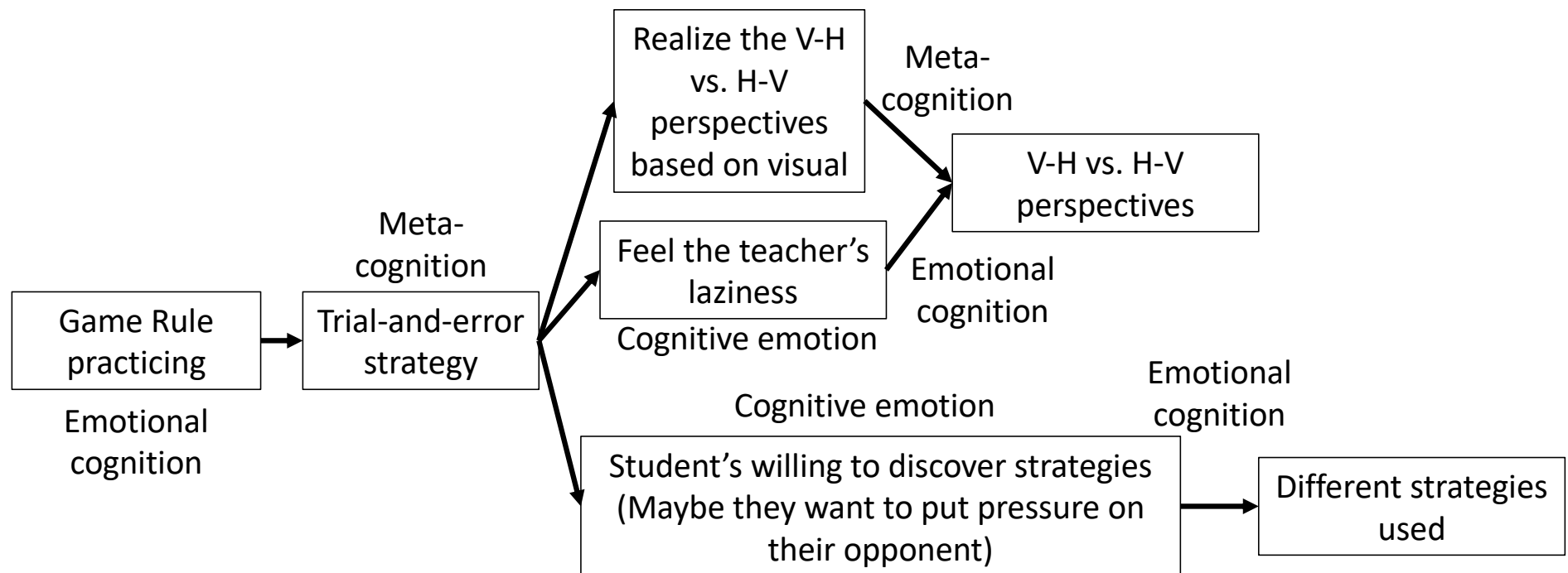
Diagnostic intervention

Holistic analysis :

1. When students check their records with teachers', they find the missing pattern (2x12) and feel surprise about their missing of the very easy pattern (2x12).
2. They wonder why they miss the pattern? They realize that because of they completely rely on 9x9 multiplication table. Is there any other strategy? Division thinking comes to their mind. They can avoid missing the pattern (2x12) with division thinking. Then they discover the new methods for the game playing. The discovery triggers their interest based on their thinking.
3. Teacher helps students to modify their thinking by questioning instead of direct instruction. Here, we can see that the surprise of finding the missing pattern is used to **trigger students' thinking** by the teacher.
4. In the discussion, the teacher not only tries to help students make sense of different thinking but also help students understand systematically via the division of the number one by one naturally.

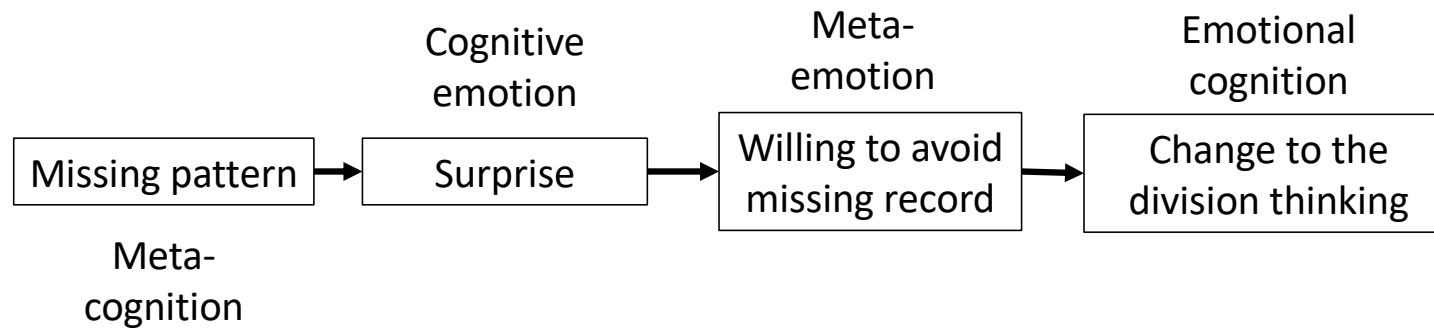
Insufficiency-based narrative analysis

The analysis referred to Hannula's (2001) four constructs



Insufficiency-based narrative analysis

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Thank you for listening.