Seminar by the Master's and Doctorate program in Mathematics Education of the Antonio Nariño University

Awareness of Insufficiency: a driving force triggering the inter-evolution of affect and cognition in learning with MGAs

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National Taiwan Normal University
2022.10.13 05:30 (Taipei time)

MGAs ?

Mathematics- Grounding Activities

A system of Generic Learning Activity

Sketch of content

Part One. The context

A national program, Just Do Math (JDM) & one of its product (MGAs)

Part Two. Modelling the learning process with MGAs

I. Enactivist approach on learning with MGAs (Yang, Lin, & Tso, 2021)

II. Modelling the learning process with MGAs

Part Three. Reflection / Discussion

A national program (2013~)

Success factors for a national Problem-driven program aimed at enhancing affective performance in mathematics learning. (Wang, Lin, & Yang, 2021)

- 🔍 National program
- 🔍 Enhancing affective performance
- 🔍 Problem-driven

Wang T.Y., Lin F.L., Yang K.L. (2021). Success factors for a national problem-driven program aimed at enhancing affective performance in mathematics learning. ZDM–Mathematics Education, 53(5), 1121-1136.

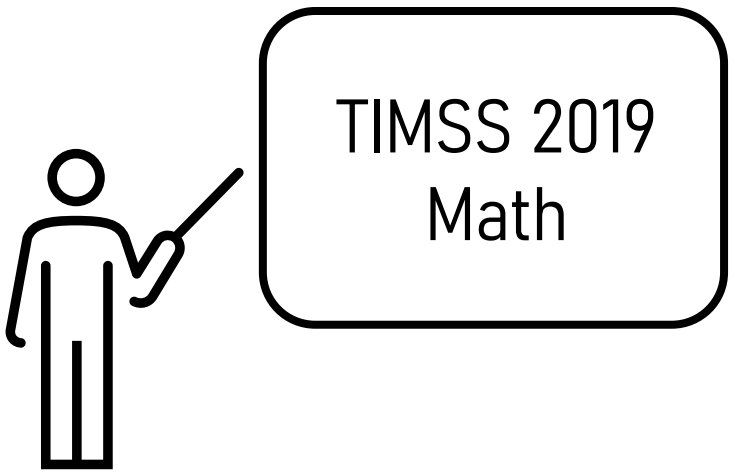
Problem-driven

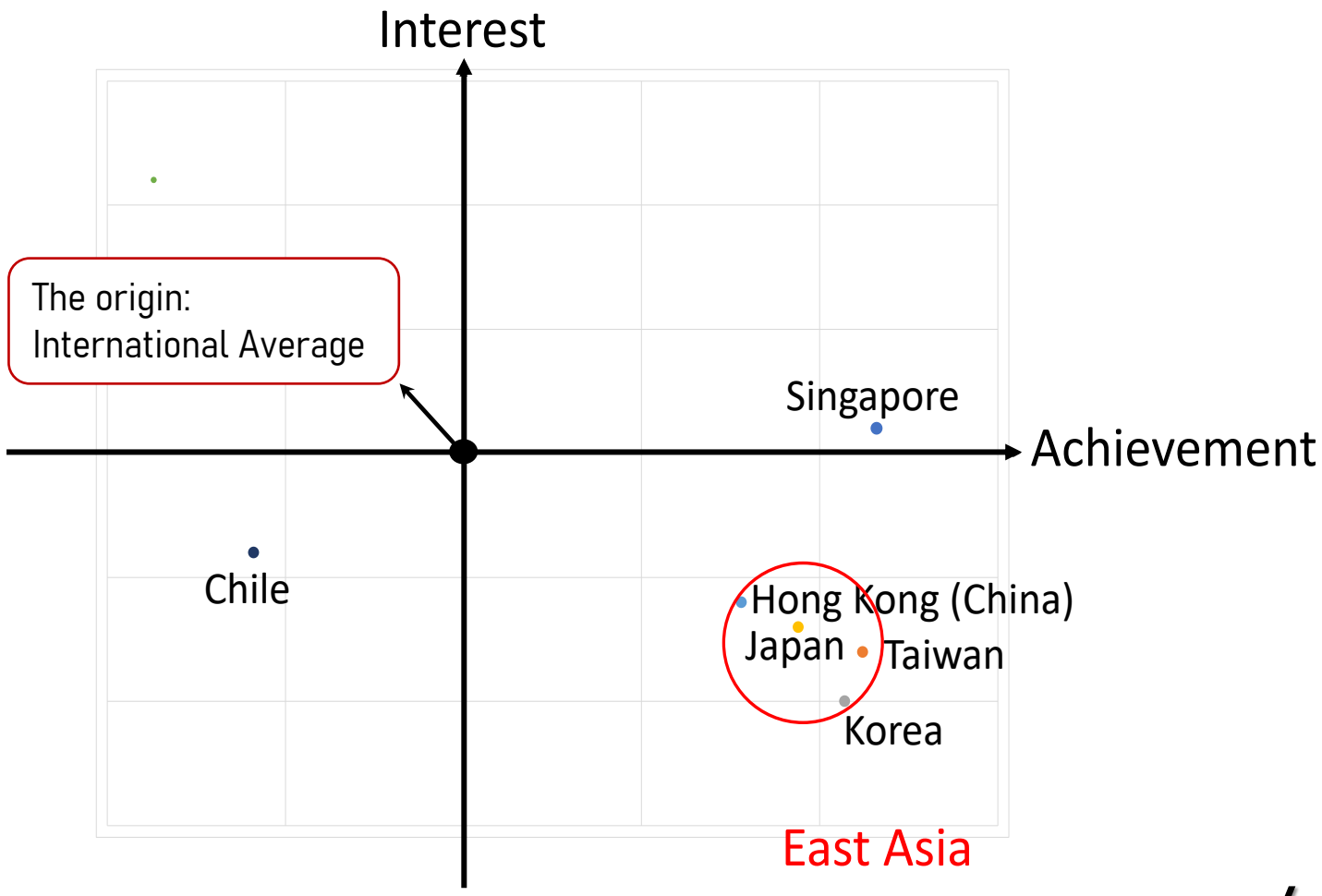
What are the national problems

in Taiwanese school mathematics learning ?

Phenomena of Taiwan students' learning

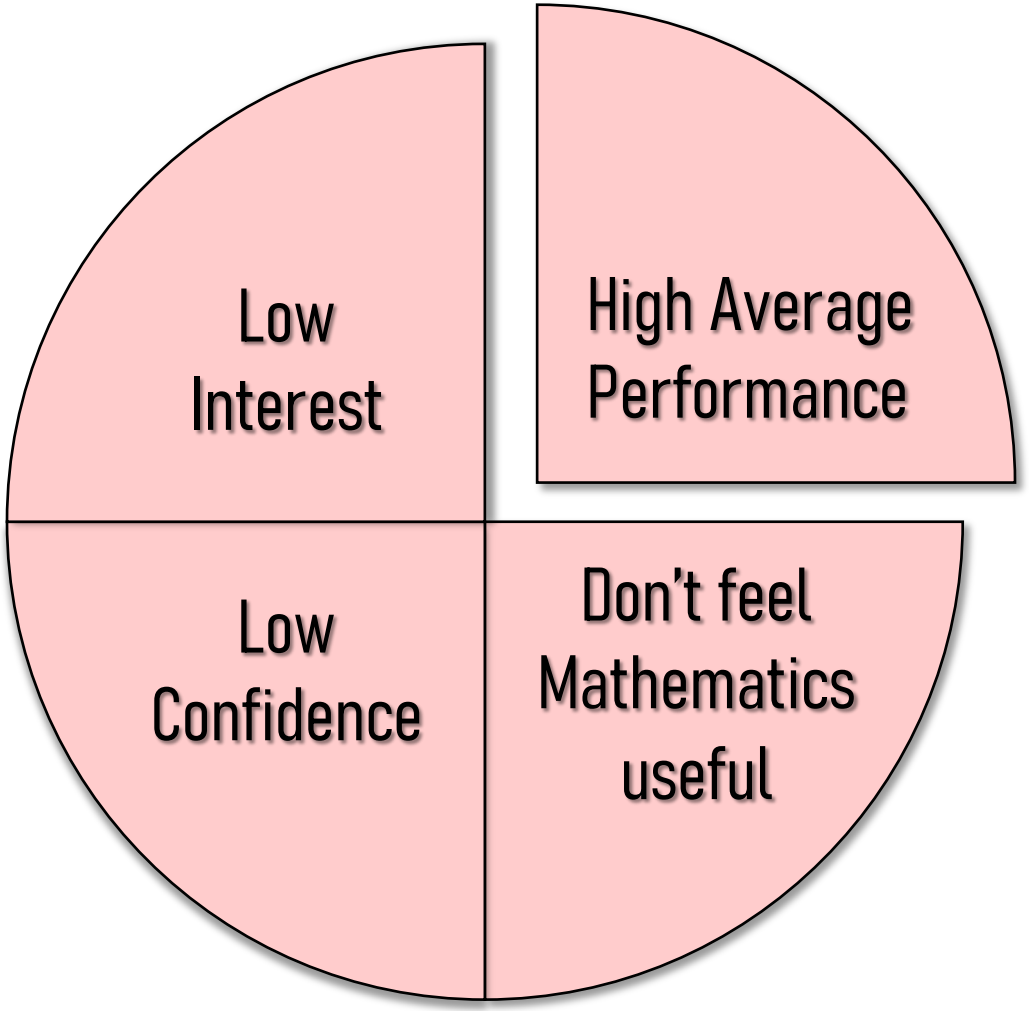
Taiwanese students' performance in TIMSS & PISA





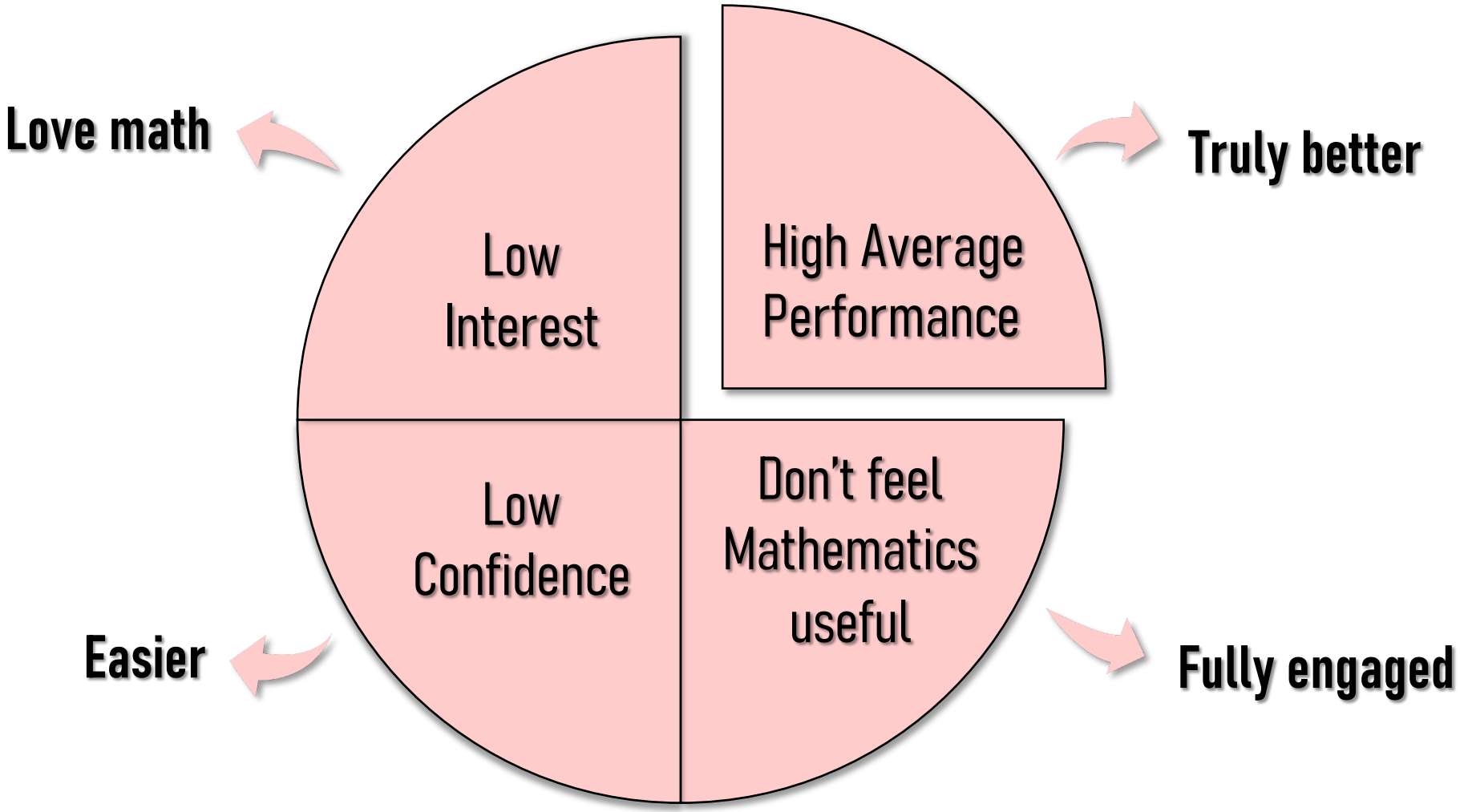
Phenomena of Taiwan students' learning

Taiwanese students' performance in TIMSS & PISA



Phenomena of Taiwan students' learning

Taiwanese students' performance in TIMSS & PISA



MGAs in JDM

A national program, Just Do Math (JDM) & one of its product (MGAs)

MGAs in JDM

The National Program : Just Do Math (JDM)

The goals of Just Do Math Program (2013~) are set to provide opportunities for students

To Love math

To learn math in an Easier way

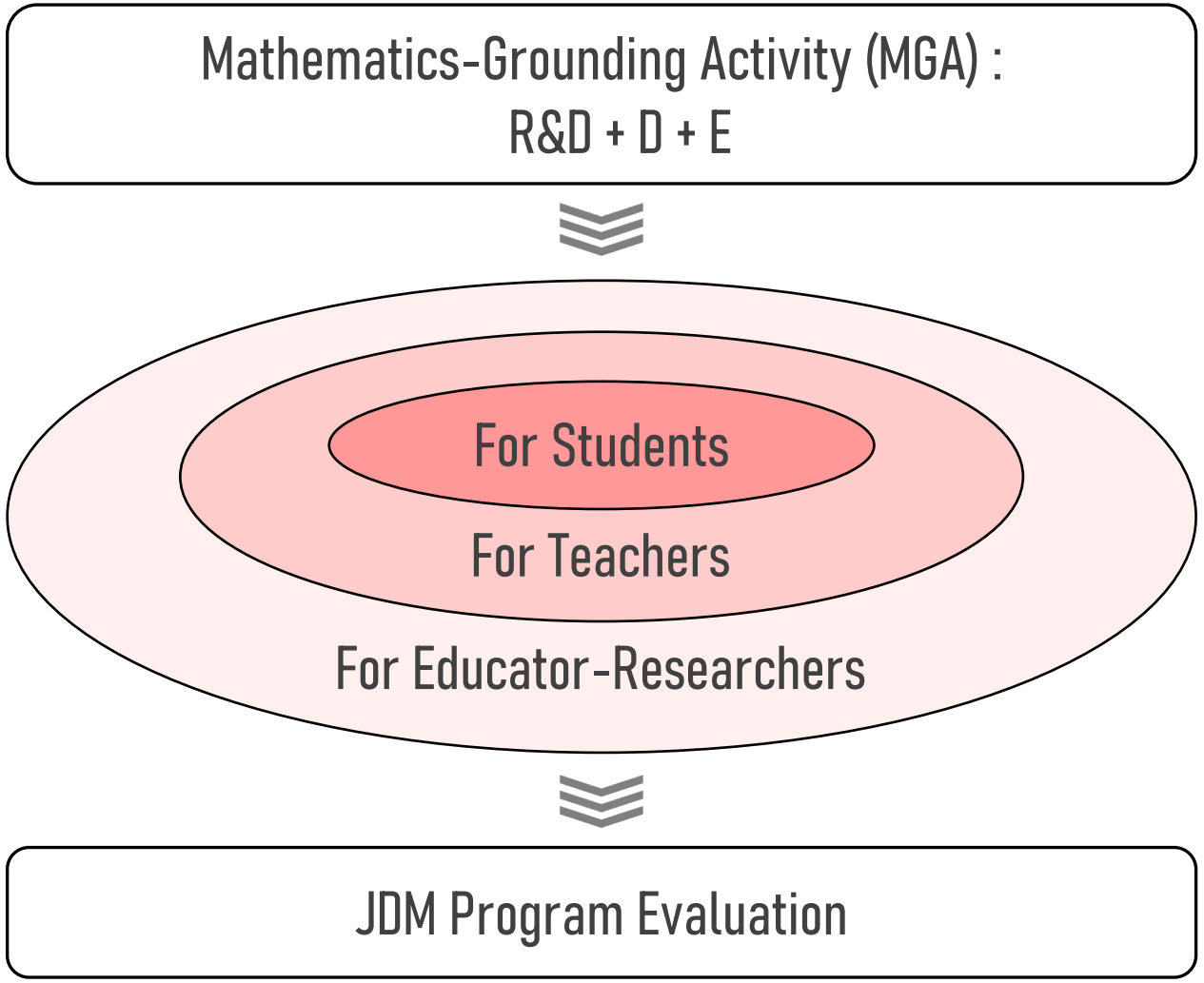
To learn it truly Better.

To Fully Engaged in learning math

The National Program : Just Do Math (JDM)

How the goals of JDM be exercised?

The learning environment in JDM



R&D: Research & Development
D: Dissemination
E: Evaluation

Settings of Students' Learning With MGAs

FUN MATH camps

(MGA camp, out of school activity)

Ongoing 2013～

There are 175 MGAs
3 videos

Re-designing

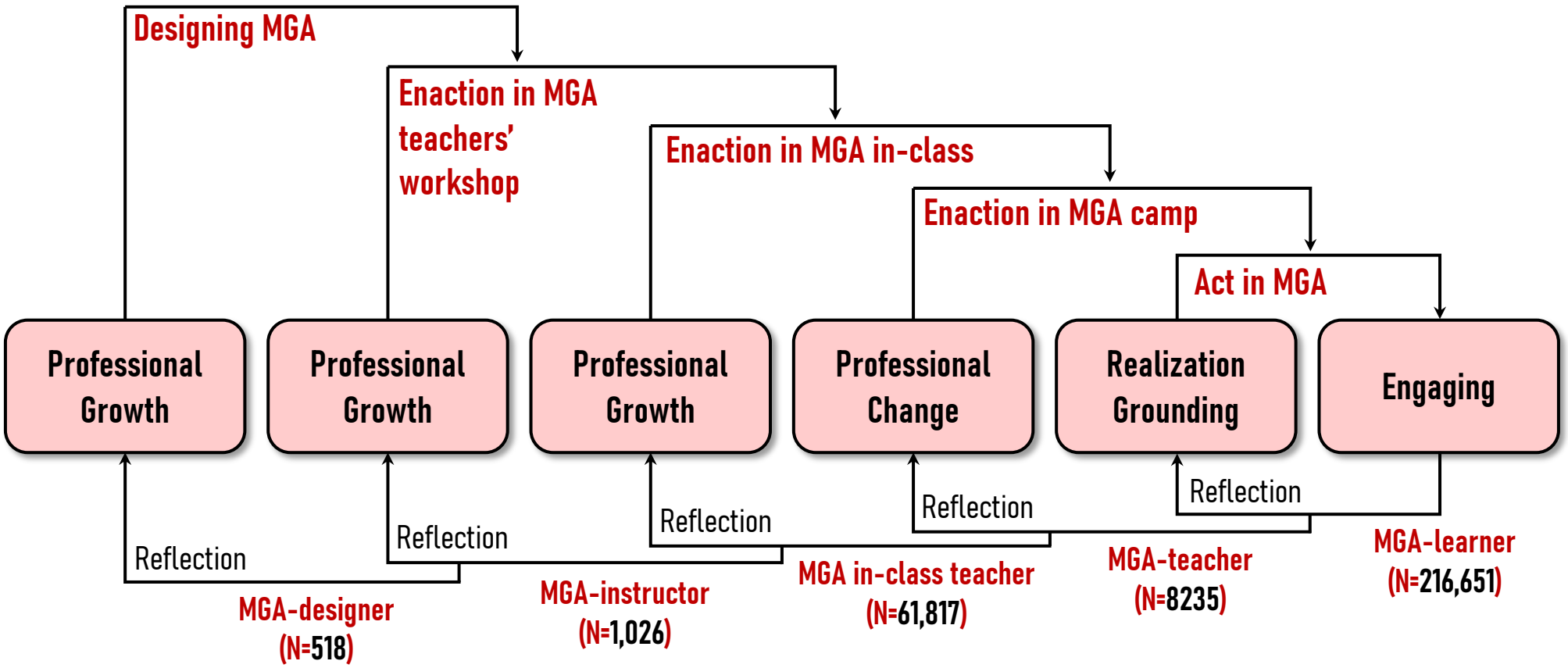
MGA-in-class

(in school lesson)

Ongoing 2017～

There are 117 MGA-in-class
52 videos

Settings of Teachers' Learning About MGAs



Teacher as learner, teacher, instructor (facilitator), designer.

Settings of Educator-Researchers' Learning In JDM

- Facilitating the designing workshops (MGA, MGA-in-class)
- ✓ Study the learning models with MGAs / MGA-in-class for students / teachers
- Co-learning among educator-researchers in various meetings
- Evaluating the JDM program
- Beyond MGAs

Initial validation of MGAs

The activity (MGAs) significantly facilitated students' cognitive and affective engagement in learning mathematics. (Lin, Wang & Yang, 2018)

Lin, F. L., Wang, T. Y., & Yang, K. L. (2018).
Description and Evaluation of a Large-Scale Project to Facilitate Student Engagement in Learning Mathematics.
Studies in Educational Evaluation, 58, 178-186

Initial validation of MGAs

The agreement percentage of 15 items on cognitive and affective engagement are all over 90% among large-scale participants.

N=8959, 2014-2015

Lin, F. L., Wang, T. Y., & Yang, K. L. (2018).
Description and Evaluation of a Large-Scale Project to Facilitate Student Engagement in Learning Mathematics.
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What is MGA?

- **Rationale :**

  Developing fundamental prerequisite mathematical ideas before they learn in regular lessons as opposed to providing remedial instruction after they encounter failure. (Prevention is better than cure. 預防勝於治療)

What is MGA?

MGAs, as generic learning activity, aim at…

1. Developing fundamental prerequisite mathematical ideas before they learn in regular lessons .
2. Constructing references for mathematical concepts through the operation of manipulable representations.
3. Increasing students' learning motivation through gamified activity.

Wang, T. Y., Lin, F. L., & Yang, K. L. (2021). Success factors for a national problem-driven program aimed at enhancing affective performance in mathematics learning. ZDM–Mathematics Education, 53(5), 1121-1136.

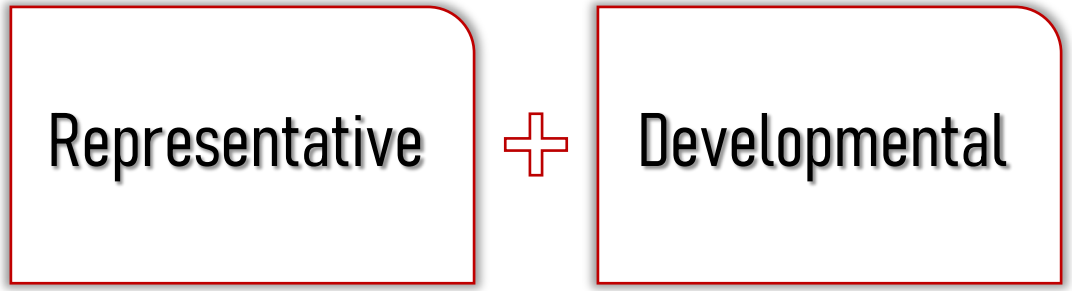
The translation of the aims of MGA

Fundamental prerequisite	⟷	Generic example
Manipulable representations	⟷	Easier
Gamified activity	⟷	Enjoyable

Wang, T. Y., Lin, F. L., & Yang, K. L. (2021). Success factors for a national problem-driven program aimed at enhancing affective performance in mathematics learning. ZDM–Mathematics Education, 53(5), 1121-1136.

The Features of Generic Learning Activities (GLA)

(Mason & Pimm, 1984; Cheng, 2000)

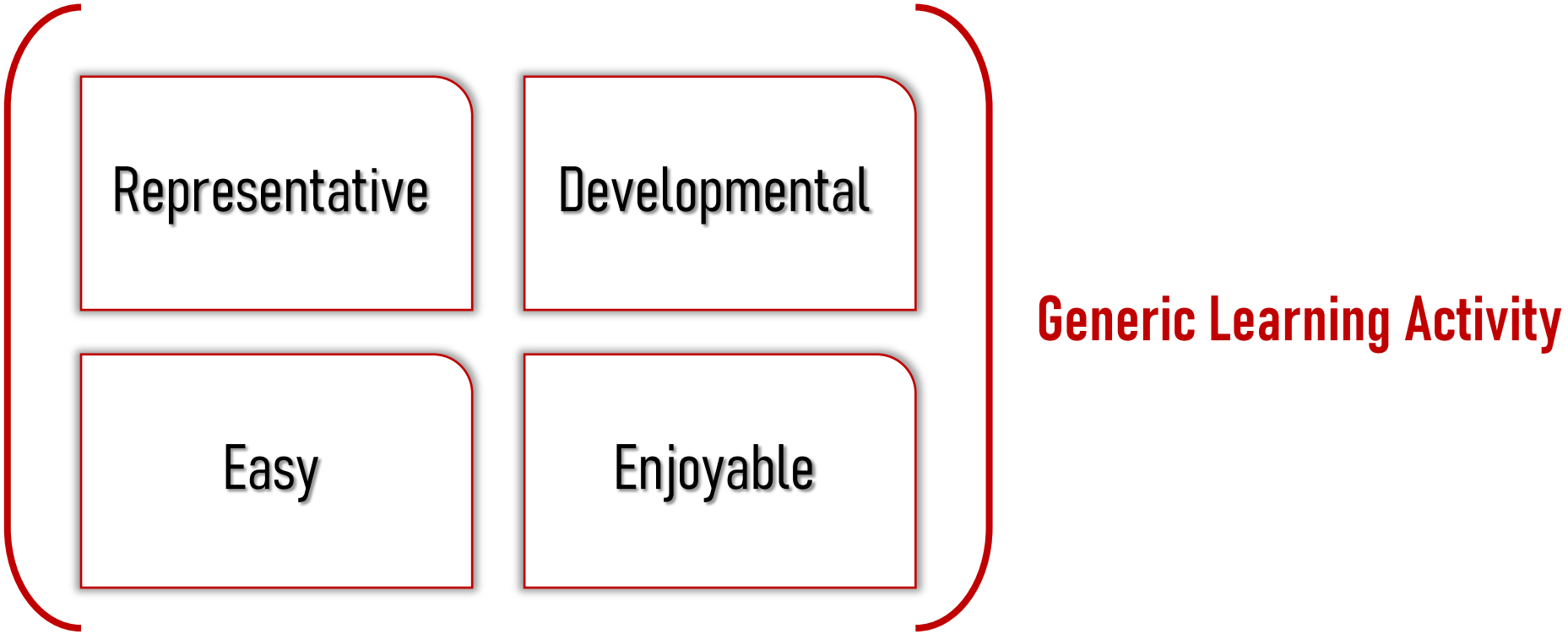


(Generic example)

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. The Asia-Pacific Education Researcher, 1-10.

The Features of Generic Learning Activities (GLA)

(Mason & Pimm, 1984; Cheng, 2000)



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Origin of MGA

How the approach of MGAs, as generic learning activity, pops out?

Origin of MGA

The initial thinking :

A Learning Trajectory of my Grandson (G2) with Rectangular Number Game (Lin & Chang, 2019)

Rectangular Number Game :

- Delivered by R. R. Skemp in a primary school, Taipei, Taiwan (1983).

Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.

Origin of MGA

Structuralist approach on mathematics curriculum development

- Key ideas: meaningful learning (Ausubel), embodied learning mode (Bruner), game (Dienes)
- Skemp is viewed as a structuralist.

Task designing model :

Manipulation-Getting a sense of-Articulation (Mason & Johnston-Wilder, 2006)

Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.

MGA : Research & Development

See Appendix

MGA-in-class : Research & Development

MGA-in-class : R & D

Four principles of designing MGA-in-class :

1. Triggering thinking principle: Triggering thinking for enhancing intrinsic learning motivation and developing positive learning attitude.
2. Sense making principle : Promoting students' active sense making continuously.
3. Co-constructing principle : Promoting discussing, thinking and co-constructing.
4. Diagnostic principle : Diagnosing conceptual understanding.

SDiME. (2019). The manual for MGA in class.

MGA-in-class : R & D

Tools for designing MGA-in-class:

1. Manipulating
2. Gamifying
3. Goal-oriented and systematic observing
4. Expressing and communicating
5. Proofing and refuting

SDiME. (2019). The manual for MGA in class.

Examples ?

I expect you have already watched the video

Rectangular Number Game (RNG)

Video : Rectangle Number Game (RNG in-class) : Rule of the game

Insert RNG video: 00:23-04:10

Linkage: https://www.youtube.com/watch?v=fImiotpvqBo

Part Two
Modelling the learning process with MGAs

Learning with MGAs

Enactivist approach on learning with MGAs (Yang, Lin, & Tso, 2021)

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. The Asia-Pacific Education Researcher, 1-10.

Analysis of RNG video

- Instruction Segments
- Features of generic learning activities

Analysis of RNG video

Instruction Segments

1. Rule of the game
2. Game playing
3. Diagnostic intervention
4. Game-based classification of numbers: considering both emotion and cognition.
   - Feeling about numbers and classifying with feelings
   - Naming the classification (informal)
5. Transformation between informal naming and formal terms
6. Structuring number classification: { 1, prime, composite numbers }
   - Distinguishing 1 from primes

Analysis of RNG video ： 4 features of the generic learning activity

Representative

1. Taking the area formula of rectangles as representation of multiplication.
2. The representation is benefit of the learning of factor, multiple and prime number concept.
3. { point, line, rectangle } is isomorphic to { 1, prime, composite number } .

Analysis of RNG video ： 4 features of the generic learning activity

Developmental

1. Classification based on scoring or not : { scoring numbers, non-scoring numbers }
2. Classification of non-scoring number : { point, line } vs. { 1, prime }
3. Structuring based on the isomorphism : { point, line, rectangle } and { 1, prime, composite number }
4. Analogues to the learning of G.C.D and L.C.M.

Analysis of RNG video ： 4 features of the generic learning activity

Easy

1. Systematic manipulative
   - Strategies for one-by-one trial-and-error
2. Classification with feeling about numbers
   - Emotions associated with different numbers is game-based.
   - Positive & negative emotions directly used as the criterion of number classification.

Analysis of RNG video ： 4 features of the generic learning activity

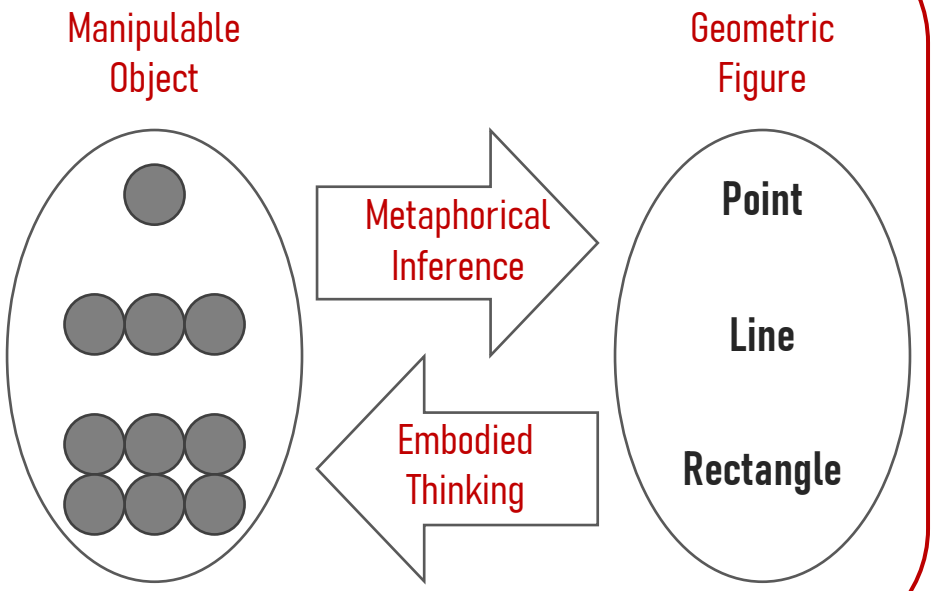
Enjoyable

1. Game playing : Prediction, expectation, testing
   - Can't be scored / scoring
   - Even scoring
2. Fresh experience
   - Classification and naming with emotion
3. Innovative naming:
   - 線點數 (Point-line numbers)、正長數 (Regular-rectangular number)
4. 1 vs. prime : Immersing in the joy of understanding the structure of number classification ｛ 1, prime, composite number ｝ after thinking.

Analysis of RNG video ： 4 features of the generic learning activity

Metaphorical Ground

Source Domain



Target Domain

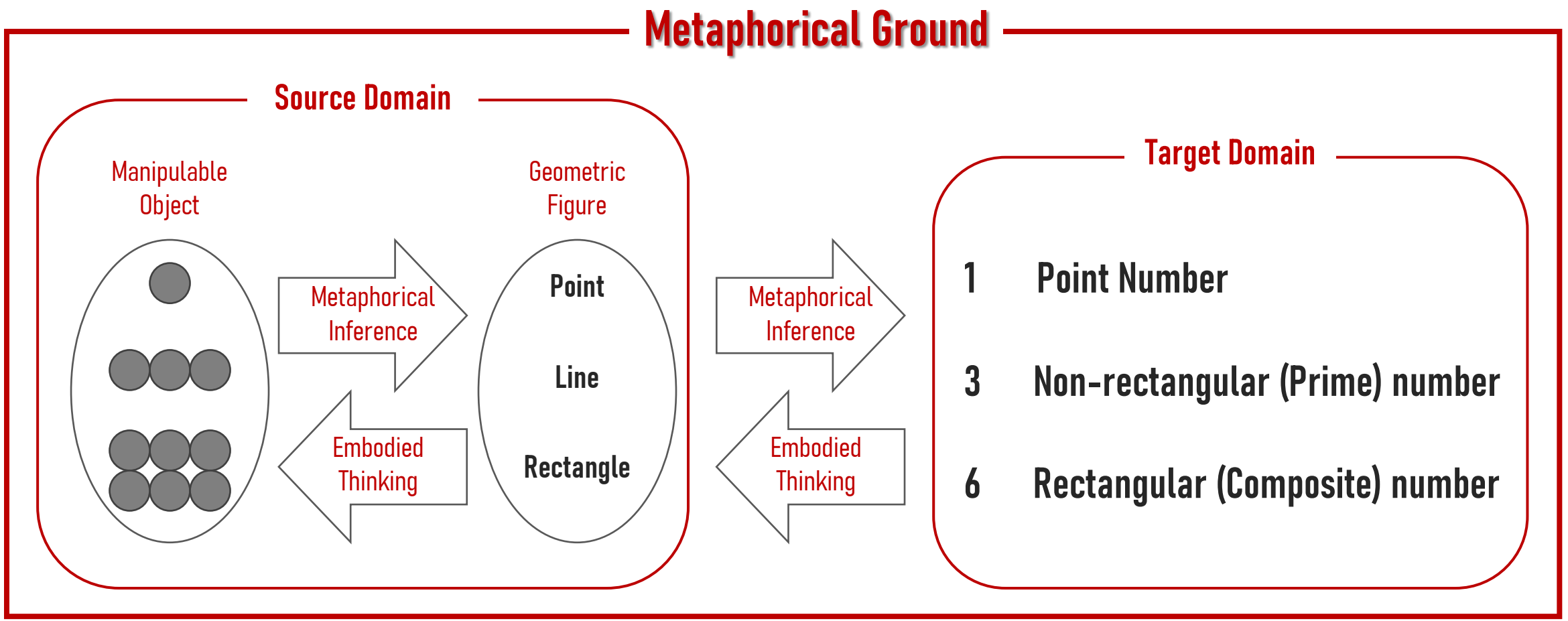
1 Point Number

3 Non-rectangular (Prime) number

6 Rectangular (Composite) number

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

Analysis of RNG video ： 4 features of the generic learning activity



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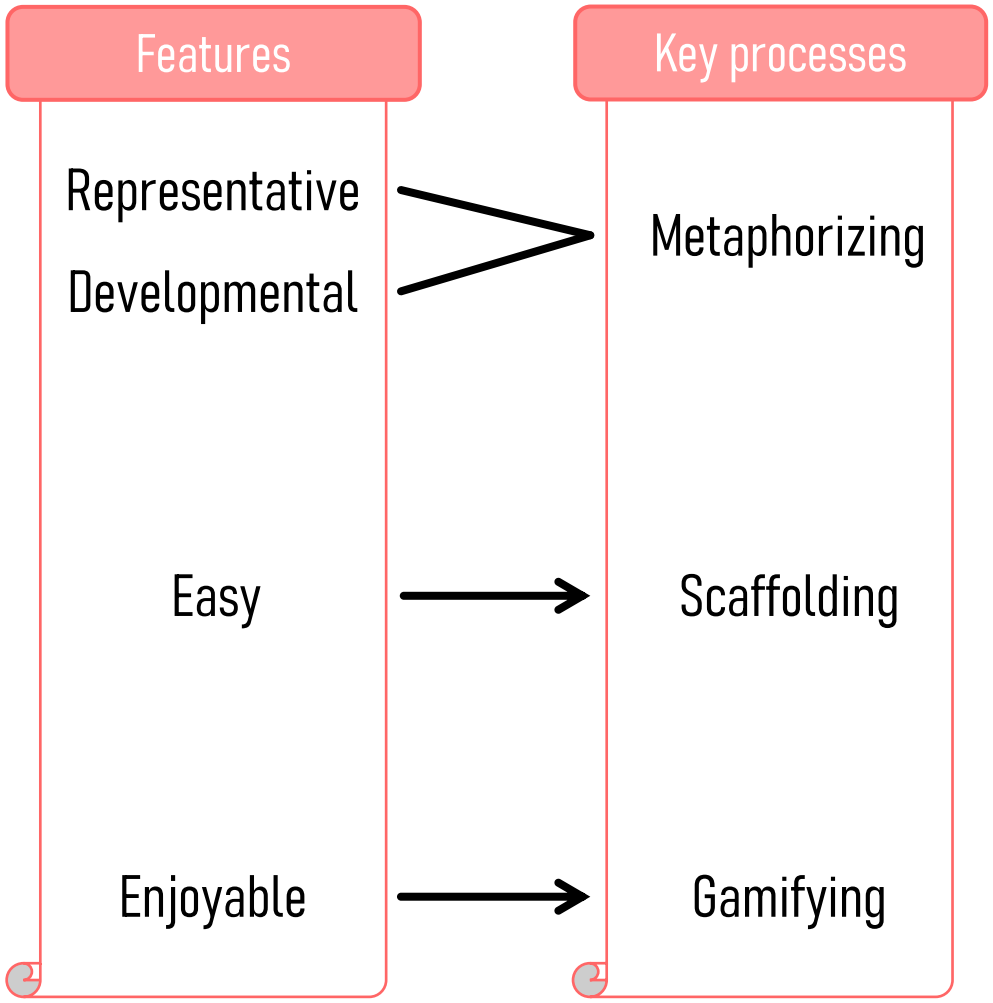
Frame of designing MGAs



Steps

- ① recognizing what the target domain of an intended mathematical concept is. (e.g. 1, prime, composite number)
- ② recognizing a source domain to embody the target domain which could be inferred as conceptual metaphors of the source domain. (e.g. points build rectangles)

- ① providing tools which can be manipulated to generate examples. (e.g. Point)
- ② providing goals which can trigger learners' awareness of the insufficiency of their emergent conceptions and enact the development of conceptual metaphors. (e.g. Diagnostic and provocative intervention)

- ① generating multiple task processes and outcomes. (e.g. The records in the game playing, the naming of the classification)
- ② generating relevant feedback as tasks evolve. (e.g. Designed worksheets)

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

Some Claims in Yang Lin, & Tso (2021)

1. Learning is the systemic interactions between learners, tasks and social contexts, and "learners' motives integrated into the evolving tasks (Yang, Lin, & Tso, 2021)."
2. Awareness of insufficiency has been noticed in the learning process with MGAs (Yang, Lin, & Tso, 2021).

Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher*, 1-10.

II. Modelling the learning process with MGAs

Modelling the learning process with MGAs (Future orientation)

學，然後知不足

《禮記・學記》 300-400 B.C.

Learning, coming to aware the insufficiency.

Modelling the learning process with MGAs (Future orientation)

A key construct of learning process (Yang, Lin, & Tso, 2021):

Awareness of insufficiency

Modelling the learning process with MGAs (Future orientation)

A key construct of learning process:

Awareness of insufficiency

Learning, coming to aware the insufficiency

(學，然後知不足)《禮記・學記》(Li Chi, Xue Chi) 300-400 B.C.

It means that one will find their insufficiencies after learning, and these insufficiencies will promote them to learn better (學無止境) (ibid.)

Modelling the learning process with MGAs (Future orientation)

An intertwining process of learning:

Inter-evolution of affect and cognition

Why the inter-evolution? It's a trend...

- Holistic psychology of mathematics education (Roth & Walshaw, 2019).
- Meta-level Classification of cognition and emotion (Hannula, 2001).
- Motivation and self-efficacy interacting in problem-solving/posing (Voica, Singer & Stan, 2020).

Modelling the learning process with MGAs (Future orientation)

How the inter-evolution of
affect and cognition be triggered?

Modelling the learning process with MGAs (Future orientation)

❓ **Hypothetical assumption:**

Would awareness of insufficiency be the driving force of inter-evolution of affect and cognition in the learning with MGAs ?

🎯 **Goal of analysis:**

Advancing learning theory focusing on the inter-evolution of affect and cognition.

Modelling the learning process with MGAs (Future orientation)

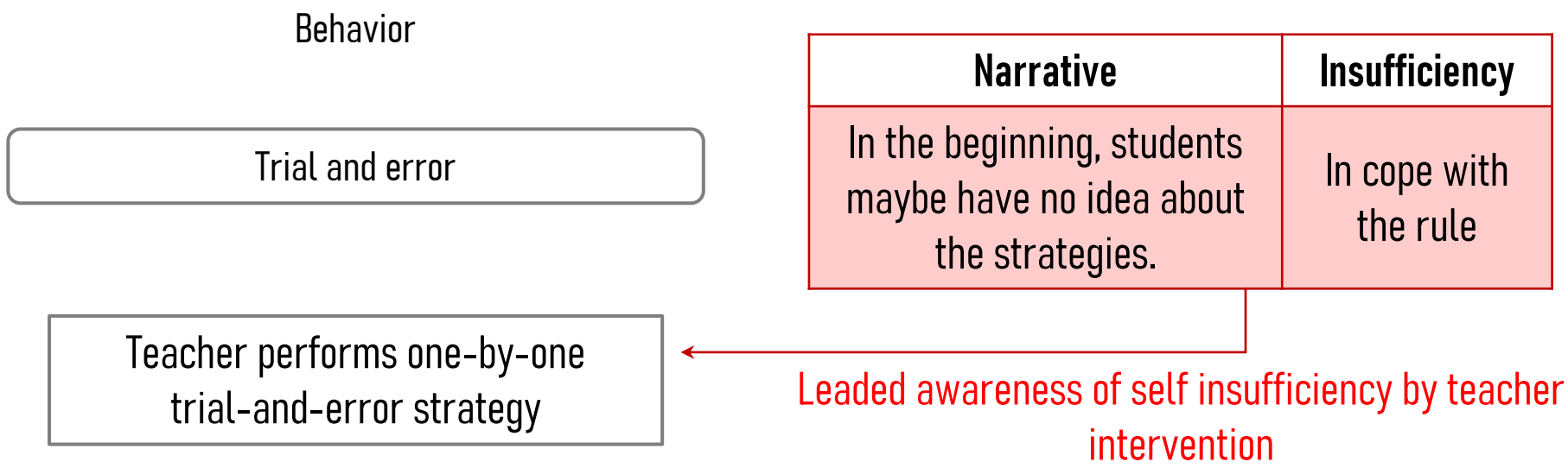
- **Methods: Narrative analysis.**
- **Framework**

1. Instruction segments (the narratives)
   - Types of awareness of insufficiency
   - Inter-transition between emotion and cognition

Insufficiency / Awareness	Self	Others
Autonomous	S-A	O-A
be Leaded	S-L	O-L

Modelling the learning process with MGAs (Future orientation)

Rule of the game

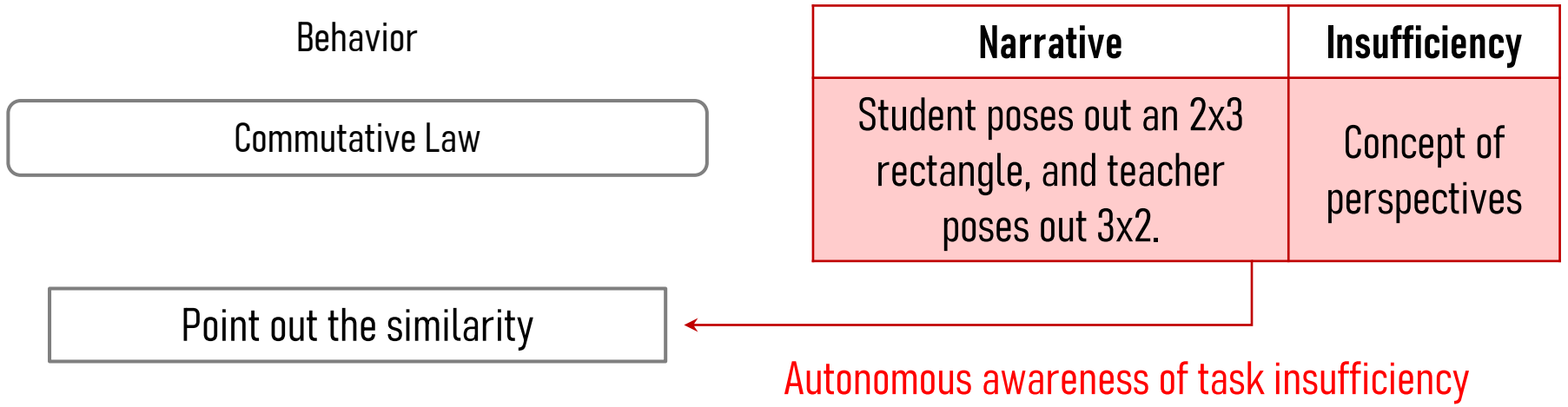


Analysis：

The teacher performs the one-by-one trial-and-error strategy to the students in order to let students feel hopeful, interest and make sense of the game.

Modelling the learning process with MGAs (Future orientation)

Rule of the game



Analysis：

1. The rectangle posed by the teacher lets students feel the teacher's laziness because they feel that the teacher only rotates the student's result. He didn't need to think. (This triggers students' body sense about the shape rotation)
2. This feeling triggers students to reject that 2x3 and 3x2 are different cognitively. (Let students make sense and build the rule)

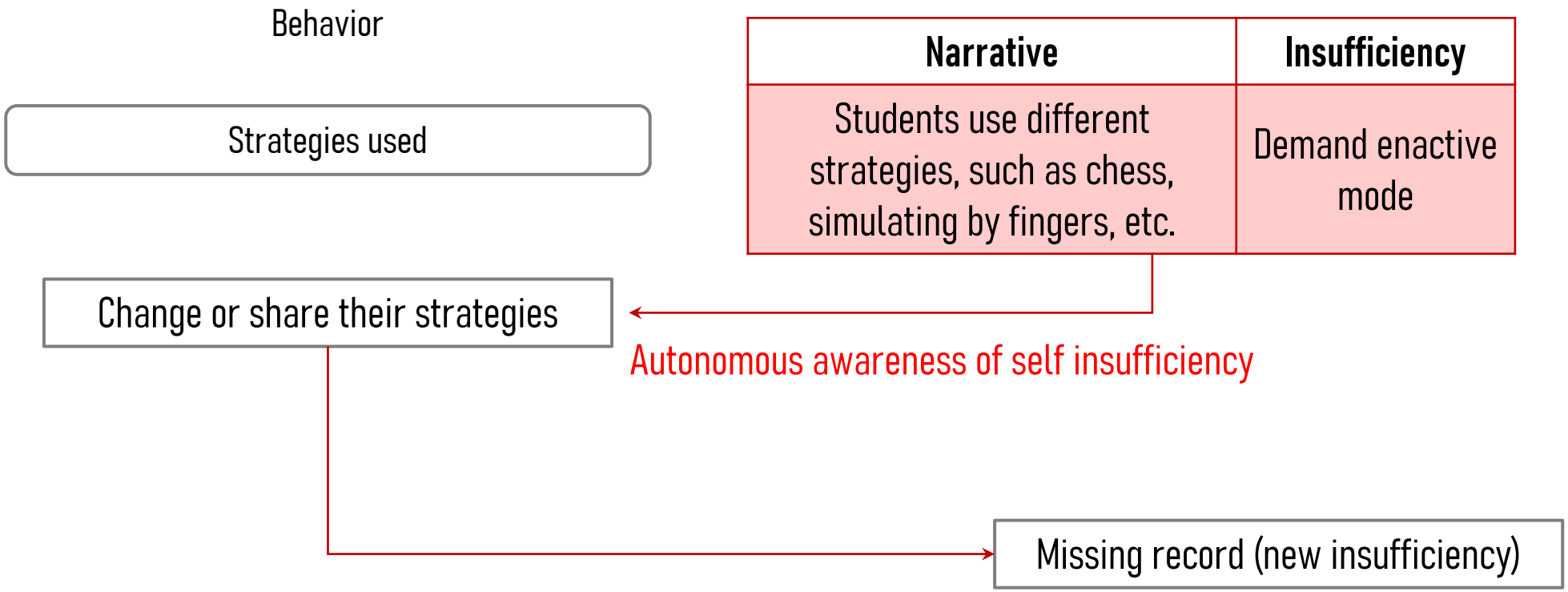
Video : Rectangle Number Game (RNG in-class) : Game playing

Insert RNG video: 04:10-05:57

Linkage: https://www.youtube.com/watch?v=fImiotpvqBo

Modelling the learning process with MGAs (Future orientation)

Game playing



Modelling the learning process with MGAs (Future orientation)

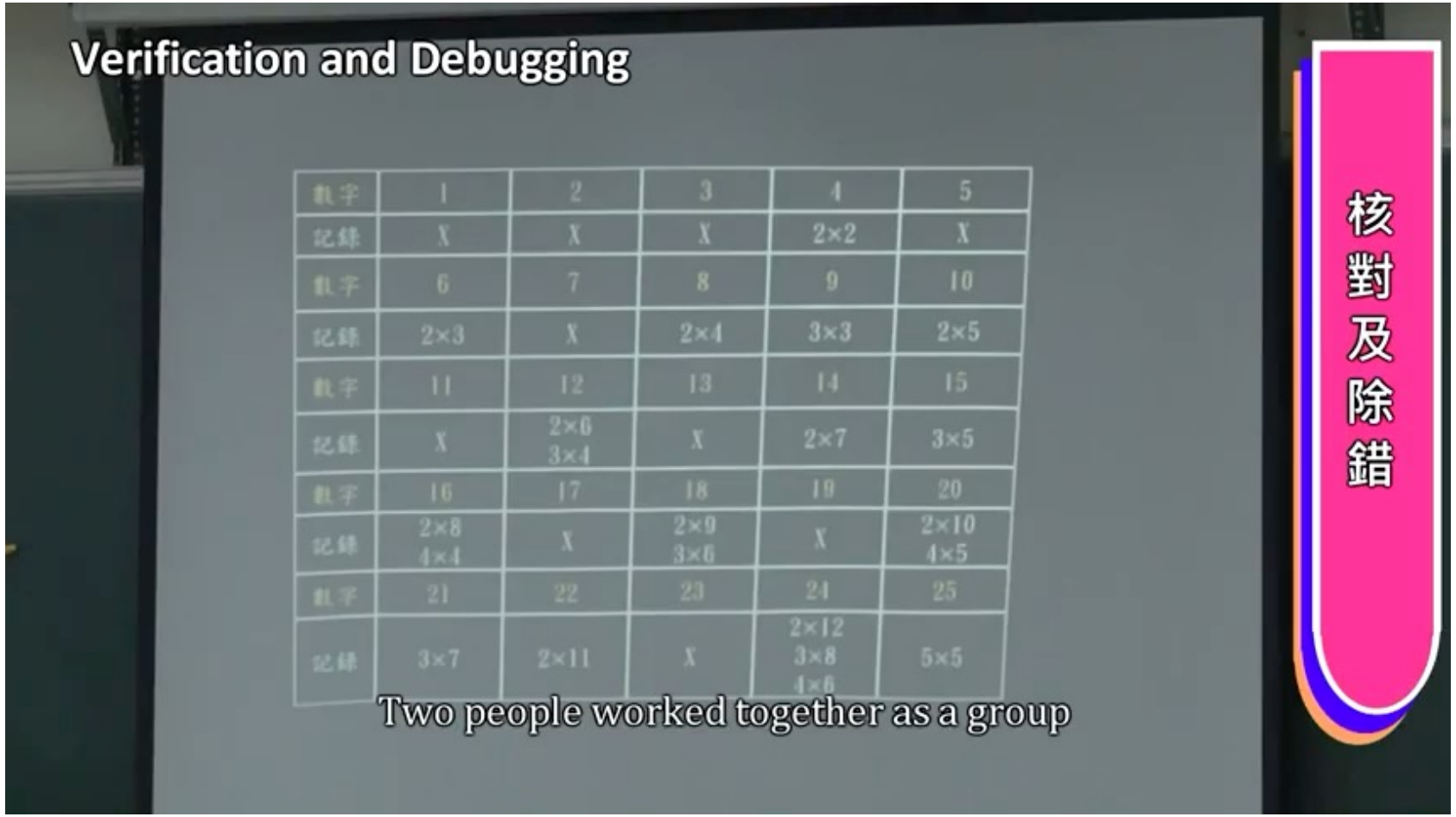
Game playing

Analysis：

1. During the game play, students use different tools such as chesses, finger simulation (enact), drawing circle (iconic mode), multiplication table.
2. Students can switch tools and can keep playing the game, so they feel easy and make sense of the game.
3. In addition, during the game play, students will also use their peers' strategies, i.e. they will share the strategies with each other. (The active co-construction between peers)
4. However, new insufficiency will also happen, such as when students rely completely on 9x9 multiplication table, they might miss the multiplication beyond 9x9.

Video : Rectangle Number Game (RNG in-class) : Diagnostic intervention

Insert RNG video: 06:13-07:41



Linkage: https://www.youtube.com/watch?v=fImiotpvqBo

Modelling the learning process with MGAs (Future orientation)

Diagnostic intervention

Behavior

Missing record

Change to the division thinking

Narrative	Insufficiency
Student miss the record 2x12 of 24.	Constrain by tools

Leaded awareness of self insufficiency by teacher intervention

Modelling the learning process with MGAs (Future orientation)

Diagnostic intervention

Analysis：

1. When students check their records with teachers', they find the missing pattern (2x12) and feel surprise about their missing of the very easy pattern (2x12).
2. They wonder why they miss the pattern? They realize that because of they completely rely on 9x9 multiplication table. Is there any other strategy? Division thinking comes to their mind. They can avoid missing the pattern (2x12) with division thinking. Then they discover the new methods for the game playing. The discovery triggers their interest based on their thinking.
3. Teacher helps students to modify their thinking by questioning instead of direct instruction. Here, we can see that the surprise of finding the missing pattern is used to trigger students' thinking by the teacher.
4. In the discussion, the teacher not only tries to help students make sense of different thinking but also help students understand systematically via the division of the number one by one naturally.

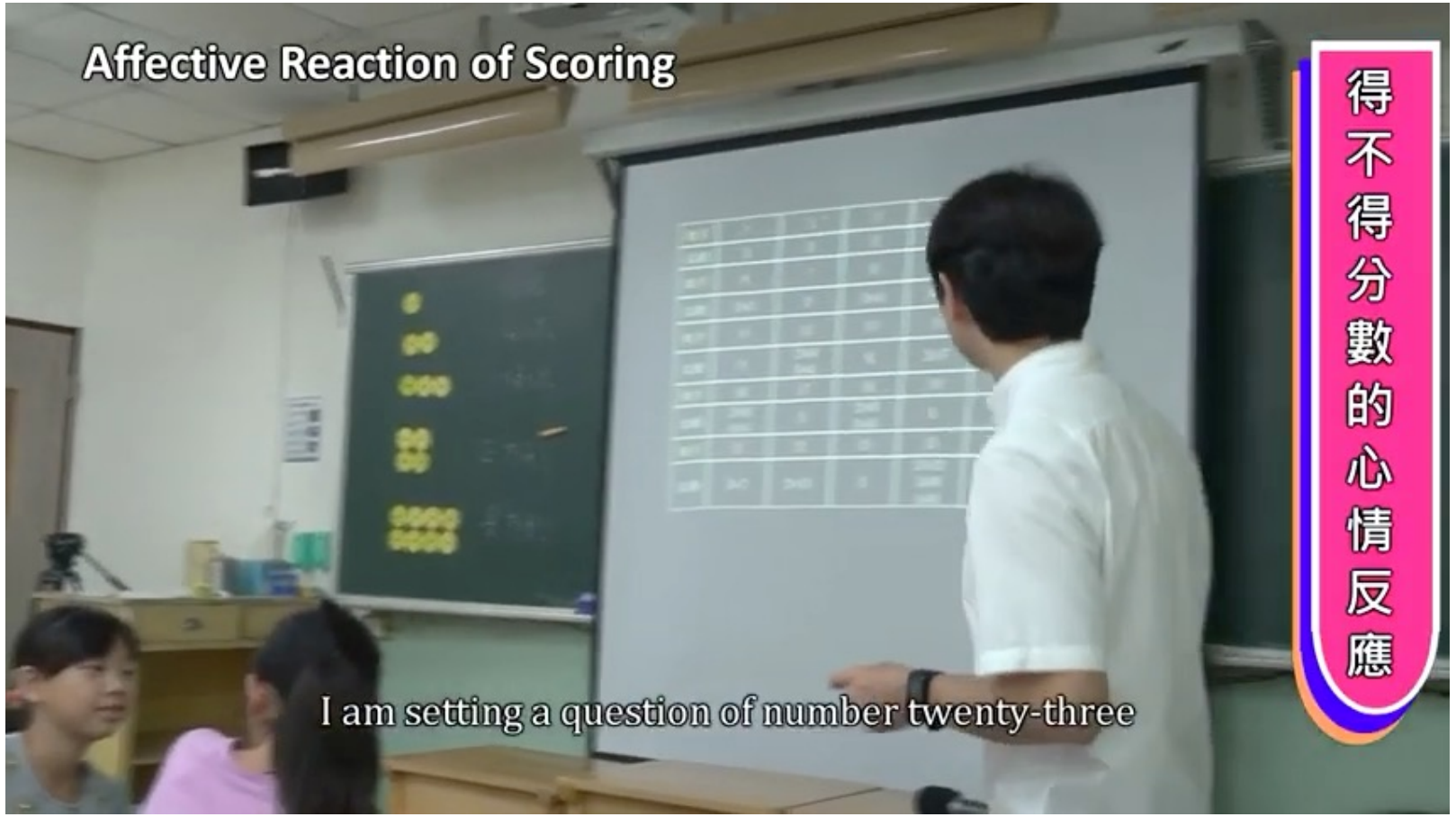
Modelling the learning process with MGAs (Future orientation)

	Content of Insufficiency	Types of awareness of insufficiency
Beginning of the game.		
Trial-and-error strategy.	In cope with the rule.	S-L
Commutative Law	Concept of perspectives	O-A
Different strategies used in the game play	Demand enactive mode	S-A
Change to the division thinking	Constrain by tools	S-L

From the beginning of the game to the diagnostic intervention, interest and sense making interwoven as a unitary learning construct.

Video : Rectangle Number Game (RNG in-class) : Feeling, classification, naming

Insert RNG video: 08:46-10:55



Linkage: https://www.youtube.com/watch?v=fImiotpvqBo

Modelling the learning process with MGAs (Future orientation)

Teachers' intervention for preparing students' inter-evolution:

Teachers' intervention	Purpose
Asking question: • If the opponent gives you the number 23, will you make efforts to form a rectangle with the number? • You cannot form a rectangle with 23, and what's your feeling about the number? How about 22?	• Retrigger students' emotion and help students link emotion and numbers. • Pave the way for students to naturally make sense of the classification of numbers.
Lead students classify numbers based on their feelings	• Trigger students' intuitive thinking based on emotion in order to let students make sense of the classification.
Lead students name the classifications with feelings or phenomenon observed	• Use students' emotion to trigger students' cognitive construction

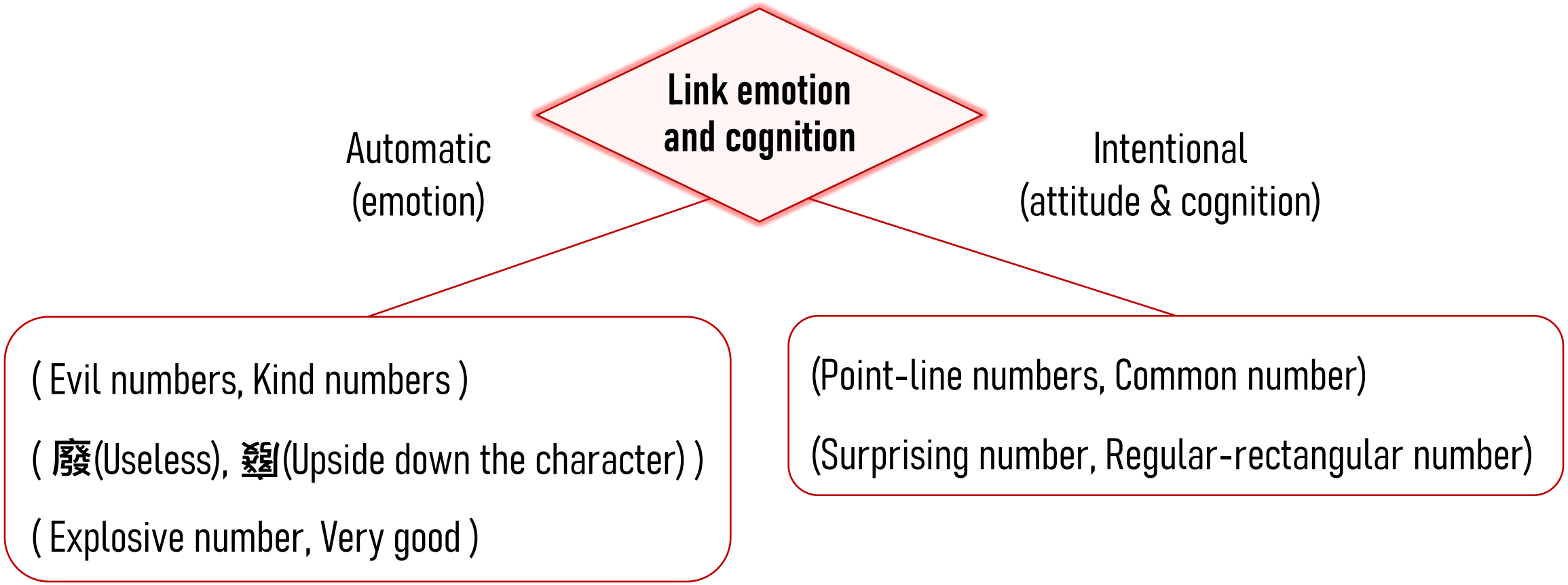
Modelling the learning process with MGAs (Future orientation)

Students' naming:

The number that cannot form rectangle	The number that can form rectangle
(Evil numbers,	Kind numbers)
(Point-line numbers,	Common numbers)
(廢(Useless number),	翹(Upside down the character))
(Surprising number,	Regular-rectangular number)
(Explosive number,	Very good)

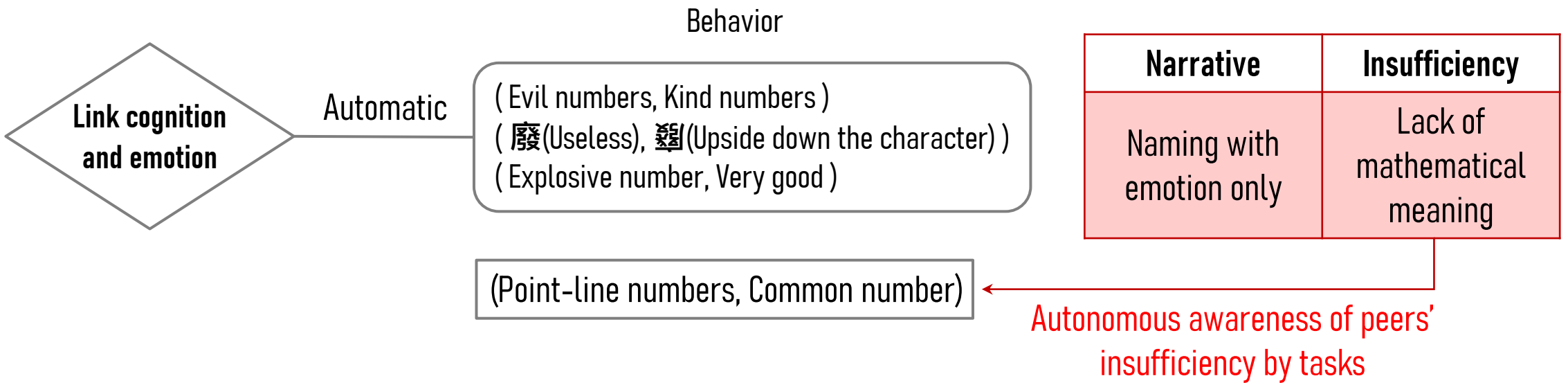
Modelling the learning process with MGAs (Future orientation)

Responses to classify numbers based on their feelings :



Automatic response vs. intentional response (adapted from Duval, 2000)

Modelling the learning process with MGAs (Future orientation)



Analysis of 線點數 (Point-line numbers) :

1. The awareness of peers' insufficiency drive students' cognitive naming with geometric figure. (A brilliant performance)
2. The feature of point-line number did not applied to the other category, it's a naming in complex.
3. Cognitive loading constrains the other category naming.

Modelling the learning process with MGAs (Future orientation)

Behavior

(Point-line numbers, Common number)

Narrative	Insufficiency
Naming with different thoughts.	Naming in complex

(Surprising number, 正長數 (Regular-rectangular number))

Autonomous awareness of peers' insufficiency

The Chinese pronunciation of 正長數 (Regular-rectangular number) is the same as the pronunciation of 正常數 (normal number)

Modelling the learning process with MGAs (Future orientation)

Socio-cultural analysis of the name 正長數 (Regular-rectangular number)

- The student meaningfully creates the new terminology "正長數(Regular-rectangular number)" and elaborate that the Chinese character of "正(regular)" is a short name for 正方形(square), and "長(rectangular)" is a short name for長方形(rectangle).
- The term 正長數(Regular-rectangular number) appears in the first time (innovative).
- In addition, the Chinese pronunciation of 正長數(Regular-rectangular number) is the same as 正常(normal), and 正常 is for normal.
- His elaboration is to avoid his peers' misunderstood of 正長數(Regular-rectangular number) as 正常數normal number.
- The naming 正長數(Regular-rectangular number) vs. rectangular number.
  - Square and rectangle are consider to be different figure for fifth graders. They don't aware the inclusive relations. This makes the naming unique and meaningful.

Modelling the learning process with MGAs (Future orientation)

Socio-cultural analysis of the name 正長數 (Regular-rectangular number)

- In the moment while he elaborates his naming, he highly appreciates his creative naming. (Here, he seems in the peak moment of learning).
- When the student immerses in the appreciation of his own naming, his creativity is triggered by peers' insufficiency, and the creativity highly triggers his self-appreciation of his naming. His cognition and affect reaches an excited states. (A state of inter-evolution of affect and cognition)

Video : Rectangle Number Game (RNG in-class) : Transformation

Insert RNG video: 11:09-12:07

Linkage: https://www.youtube.com/watch?v=fImiotpvqBo

Modelling the learning process with MGAs (Future orientation)

Behavior

(Point-line numbers, Common number)

Narrative	Insufficiency
1 and prime are in the same category	No scoring, so no motives to distinguish 1 from prime

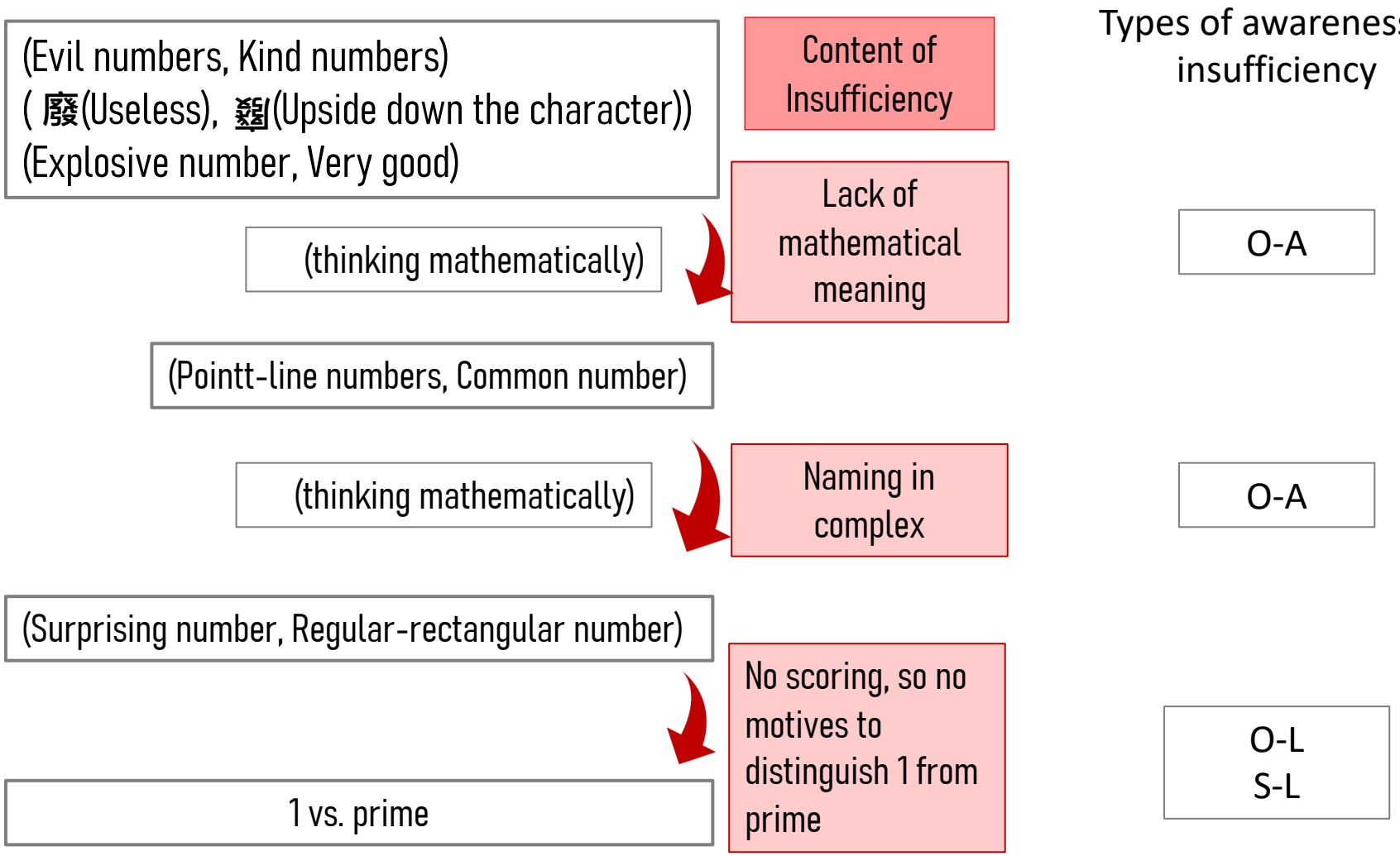
Point vs. line

Leaded awareness of game-based insufficiency by teacher intervention

Analysis:

1. During the game, due to 1 and prime are no scoring number, so no motives for distinguishing.
2. After be leaded by teacher's provocative question, a student easy to distinguish 1 from prime based on the different geometric objects.

Modelling the learning process with MGAs (Future orientation)



The whole class not only share the mathematics meaningful naming (Point-line numbers, Regular-rectangular number), but also has the potential to complete the nature number classification {1, prime, composite number}.

Modelling the learning process with MGAs (Future orientation)

Table. Records in RNG video

Insufficiency / Awareness	Self	Others
Autonomous	S-A * 1	O-A * 3
be Leaded	S-L * 3	O-L * 1

Evidence (S-L, O-A, O-L) point to the connection with Vygotskian theory of social constructivism

Prime number: Scientific concept development

Adapted three phases of Vygotsky's Concept development	Naming in RNG	Social interaction X task evolution	
		Phenomena	Insufficiency
Syncretism	(Evil numbers, Kind numbers) (廢(Useless), 鼈(Upside down the character)) (Explosive number, Very good)	Naming with game-triggered emotion directly	Lack of mathematical meaning
Naming in complex (Thinking in complex)	(Point-line numbers, Common number) (Surprising number, Regular-rectangular number)	Naming with different thoughts	Naming in complex
	(Point-line numbers, Common number)	1 and prime are in the same category	No scoring, so no motives to distinguish 1 from prime

Scientific concept

{ 1, prime, composite number }

formal definition of prime number

Develop the general and abstract concept

Generalization (?)
Abstraction (?)

Observation on awareness of insufficiency drives the inter-evolution of affect and cognition

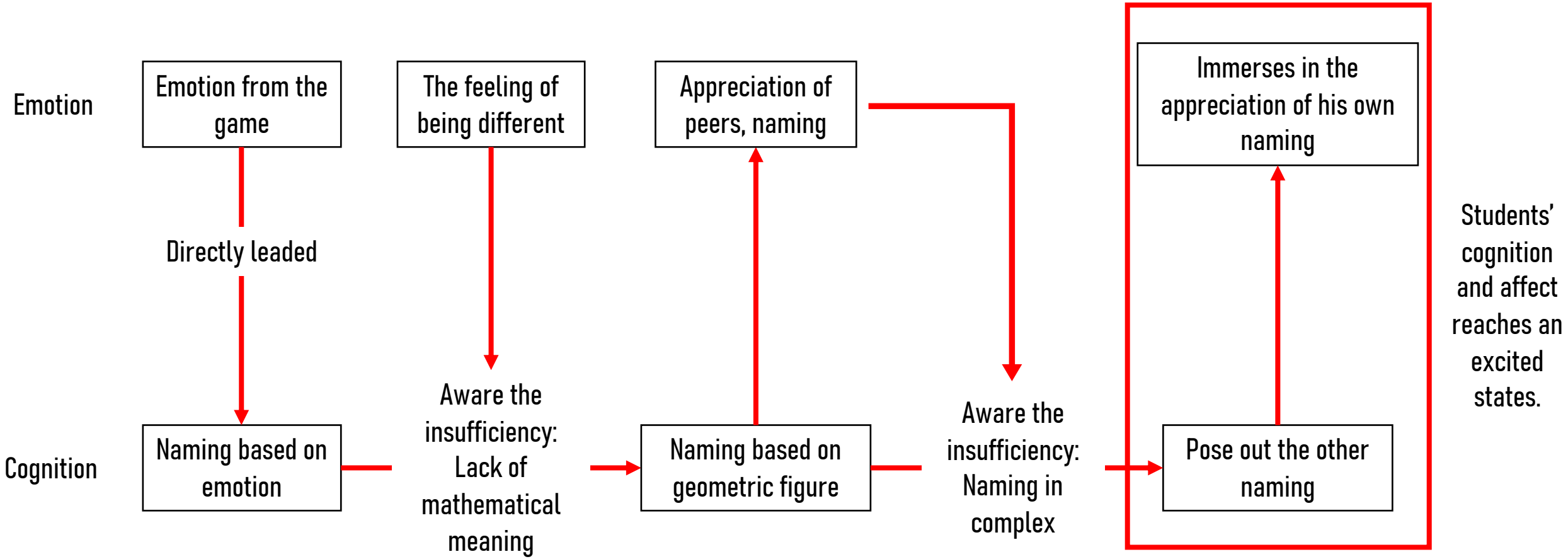
Instruction segment	Awareness of insufficiency	The inter-transition of affect and cognition
Rule of the game	Concept of perspectives	Cognition phenomenon (2x3 and 3x2) triggers students' emotion (feel teacher's laziness) to let students make sense of the commutative law.
Game playing	Demand enactive mode	Students can switch tools and can keep playing the game, they feel easy and make sense of the game.
Diagnostic intervention	Constrain by tools	When students check their records with teachers', they find the missing pattern (2x12) and feel surprise about the missing of the very easy pattern (2x12), and the surprise of finding the missing pattern is used to trigger students' thinking by the teacher.

Observation on awareness of insufficiency drives the inter-evolution of affect and cognition

Instruction segment	Awareness of insufficiency	The inter-transition of affect and cognition
Game-based classification of numbers: considering both emotion and cognition	Naming in complex	When students name the classification, they first mainly uses emotion to name because that their classification is based on their emotion, so they directly name with emotion. Here, the cognitive activity, naming, is leaded by students' emotion. The task constrain students' naming. But in the moment, one of the students see peers' insufficiency that their namings are not mathematically meaningful, so he uses the geometric figure to name. Here, maybe he tries to perform his specialty from others. The feeling let him name with the geometric figure. The naming (Point-line numbers, Common number) let students appreciate about it. After the naming is posed out, another students poses out the naming (Surprising number, Regular-rectangular number). Here, we can see that the appreciation of students trigger his naming in complex. In addition, the student immerses in the appreciation of his own naming, his creativity is triggered by peers' insufficiency, and the creativity highly triggers his self-appreciation of his naming. His cognition and affect reaches an excited states.

Observation on awareness of insufficiency drives the inter-evolution of affect and cognition

The inter-evolution of students' affect and cognition in naming in complex

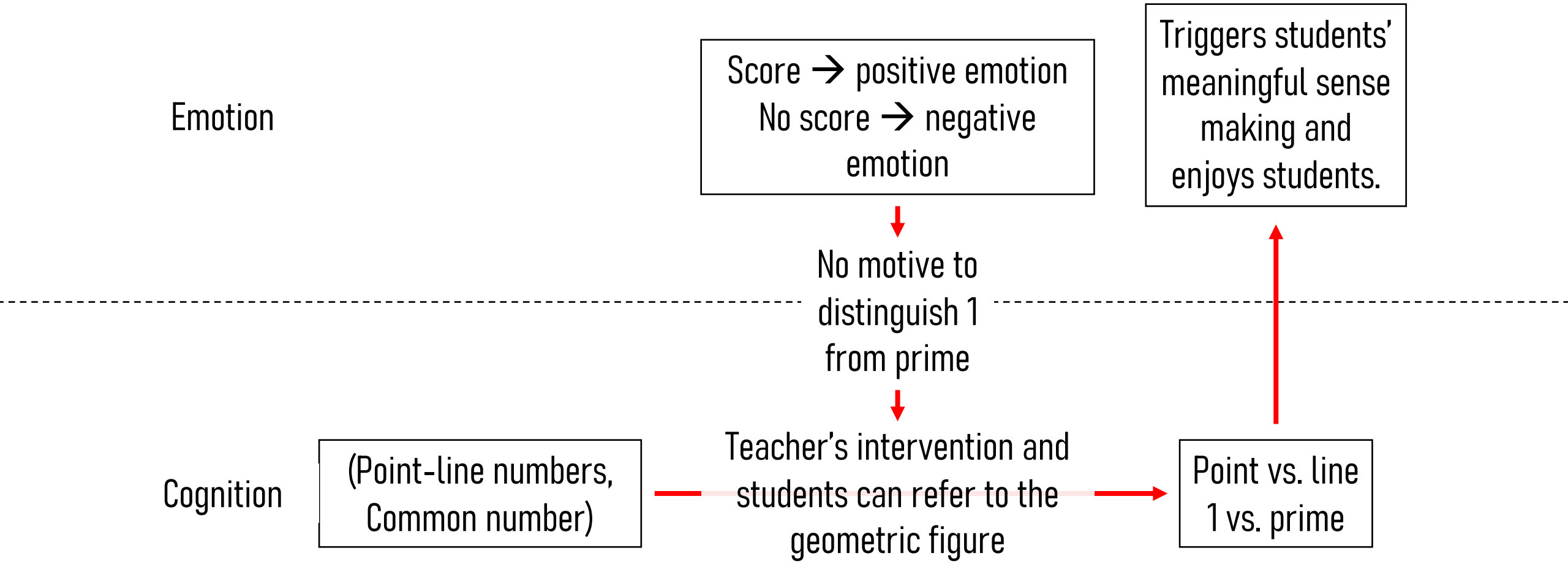


Observation on awareness of insufficiency drives the inter-evolution of affect and cognition

Instruction segment	Awareness of insufficiency	The inter-transition of affect and cognition
Distinguishing 1 from primes	No scoring, so no motives to distinguish 1 from prime	During the game play, 1 and prime cannot score, which upset students. Therefore, when students classify the number with emotion, they don't need to distinguish 1 from prime. Here, we can see that the insufficiency is caused be the inter-transition of affect and cognition in the game play and classification. Hence, teacher's intervention triggers students' motives to distinguish point from line in order to understand the difference between 1 and prime. Why students can distinguish 1 from prime? Because the teacher's question is that "what's the difference in point-line number?" The question leads students to see the difference based on the reference of geometric figure. This lets students meaningfully distinguish 1 from prime, which enjoys students.

Observation on awareness of insufficiency drives the inter-evolution of affect and cognition

The inter-evolution of students' affect and cognition in distinguishing 1 from primes



Results of the modelling

- Interest and sense making are interwoven as a holistic unitary learning construct, i.e. Interest $*$ sense making.
- Productive disposition and understanding and competencies are also interwoven as a holistic unitary learning construct.
- Learning is a cyclic self-integrative process. It starts from the holistic unitary of interest and sense making and then different kinds of awareness of insufficiencies will pave the way to nurture students' productive disposition and understanding. In the process of paving the way to the integration of productive disposition and understanding, the holistic unitary of interest and sense making happens everywhere. It's also integrated in the holistic unitary of productive disposition and understanding, and thus forms a cyclic self-integrative process.

Results of the modelling

- Awareness of insufficiency have been observed in the RNG video for triggering the inter-evolution of affect and cognition, which supports the progressive learning. Further empirical study to verify the observation above in variety of instructions are needed.
- Compared with Vygotsky's thoughts, learning happens in social interaction, but awareness of insufficiency not only helps students learn under the social interaction between teacher-students and among peers. In addition, learning also happens when the tasks evolves. Handling the tasks, students interact with the tasks, and the game-based tasks also serve the insufficiencies to promote students' learning. The learning opportunities provided by tasks evolve could be viewed as one type of social interaction.

Results of the modelling

- Awareness of insufficiency drives the learning inter-evolved with affect and cognition. We can observe the positively developing inter-evolution of affect and cognition in the settings of MGAs, and the continuously positive development of the inter-evolution of affect and cognition steps forwards to the competencies interwoven with productive disposition and understanding.
- Encouraging all students' self idea and providing representation opportunities are two strategies used in JDM to sustain the inter-transition of affect and cognition.

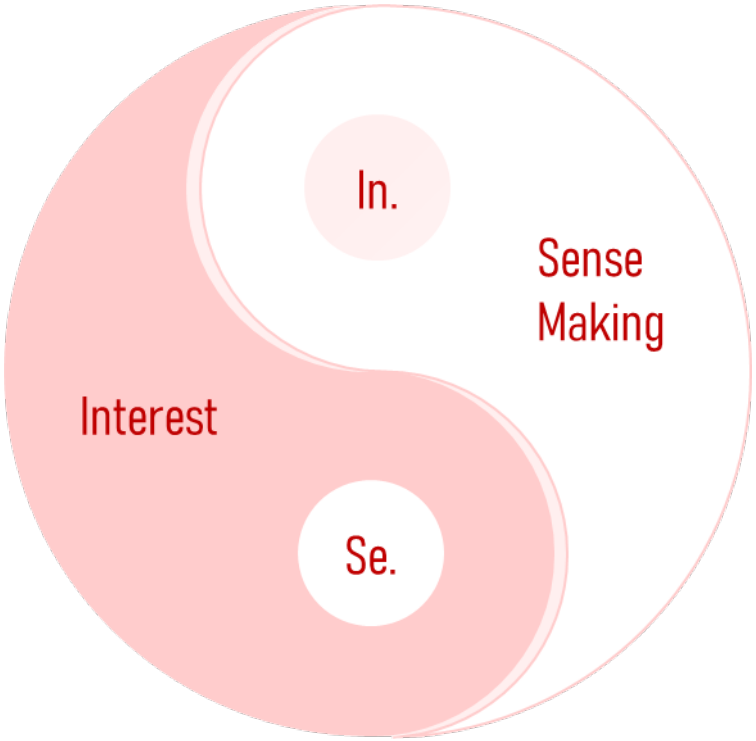
The learning process can be termed as

A Holistic Model of Learning Mathematics

and modelling as the following representation.

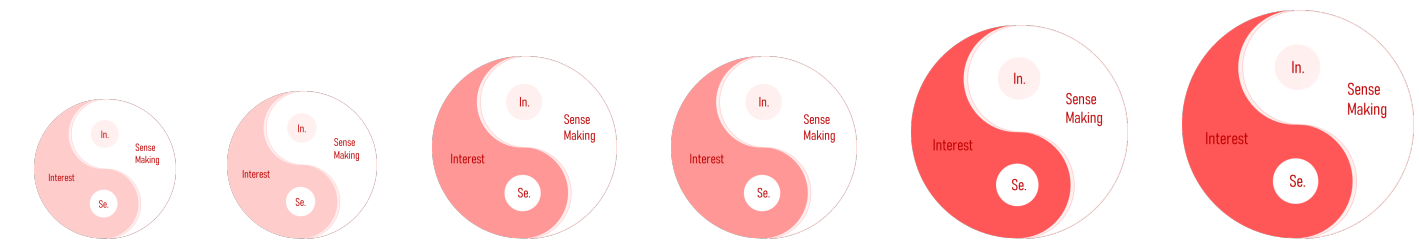
A Holistic Model of Learning Mathematics

Starter



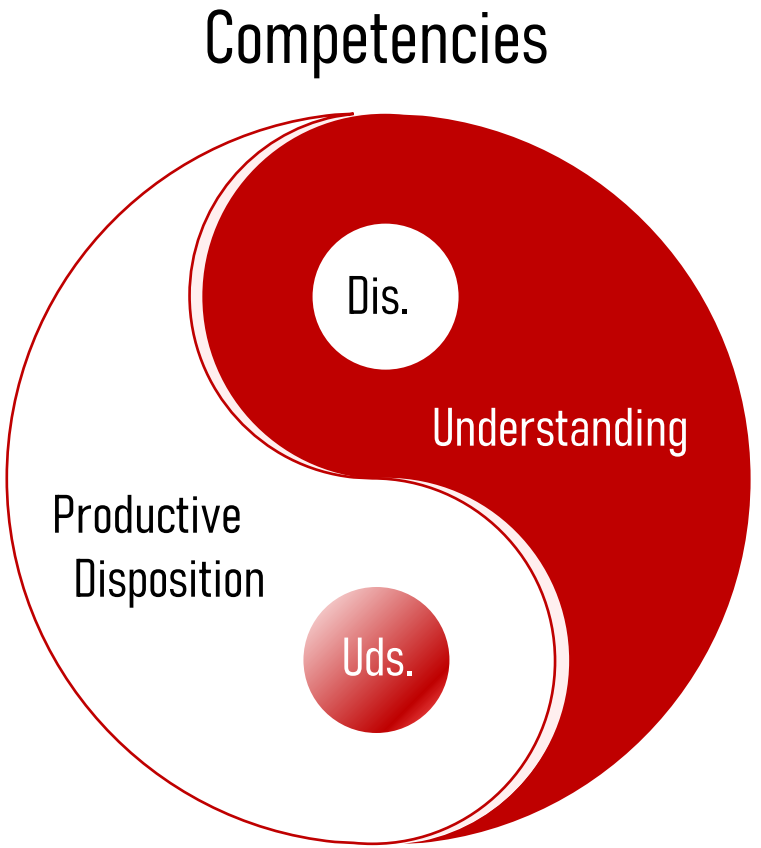
A Holistic Model of Learning Mathematics

Awareness of insufficiency drives the learning process



Starter

Sustaining the inter-transition of affect & cognition



Part Three
Reflection / Discussion

Reflection / Discussion

Reflection

1. Classification and comparison are two major types of human activity for generating mathematics.

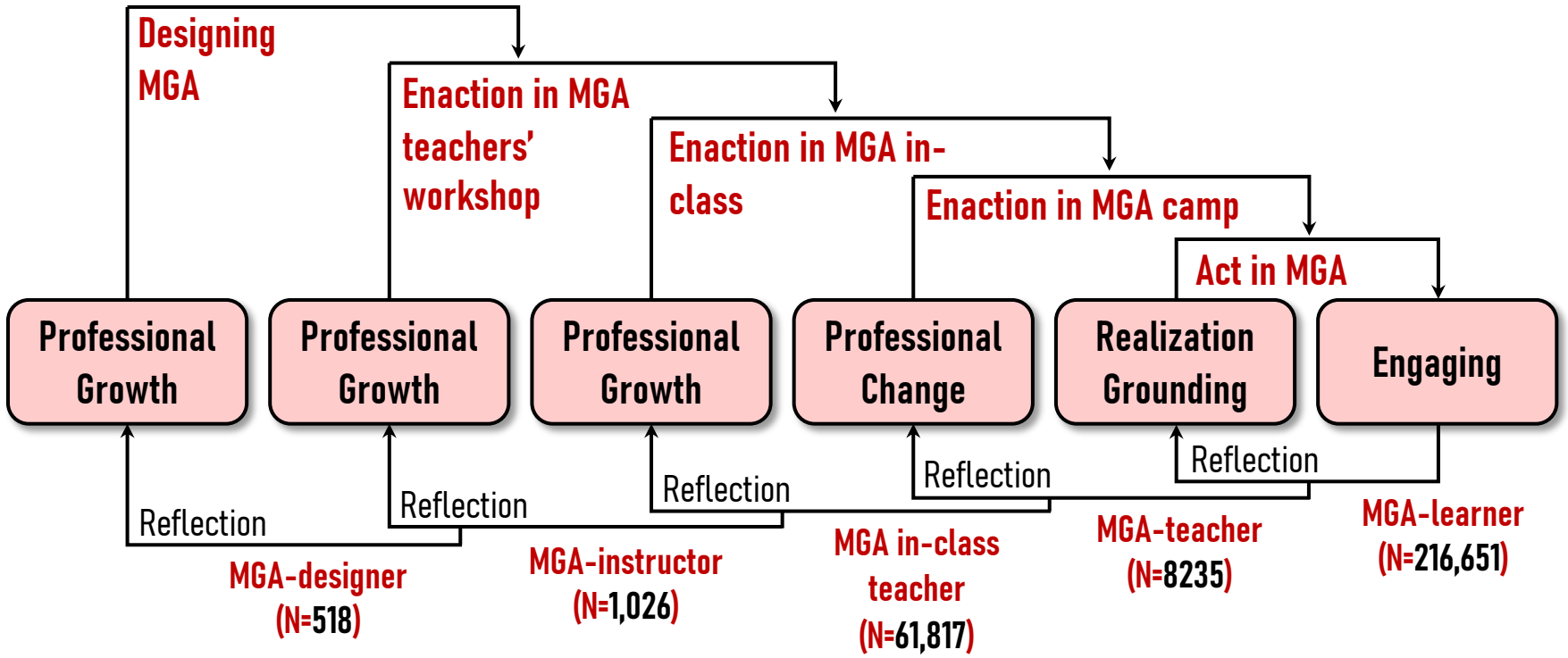
   The classification of nature numbers promoted in RNG video is that students' emotions about numbers are used as the means of thinking, medium of metaphorical inference.

2. MGA provides learning opportunities for developing students' ZPD in Vygotskian theory. E.g. 1 vs. prime number and the isomorphism between geometric structure of {point, line, rectangle} and number structure of {1, prime, composite numbers}, so that the generalized and abstract scientific concept of prime number could be developed based on such understanding.

Reflection / Discussion

Agenda for discussion

1. Can the learning process model be generalized to teachers' learning?



Role of teacher : learner, teacher, instructor, designer.

Reflection / Discussion

Agenda for discussion

2. How you value the frame of designing MGAs (Yang, Lin, & Tso, 2021) for your own innovative designing of learning activity?

Acknowledgement

I would like to thank Tsung-Ju, Wu for preparing the PPT of this conference and Jhao-Jie, Jheng as art editor of the PPT.

Content editing: Tsung-Ju, Wu.

Art editing: Jhao-Jie, Jheng.

Articles published in JDM

- Students' learning:
  - Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.
  - Yang, K. L., Lin, F. L., & Tso, T. Y. (2021). An approach to enactivist perspective on learning: Mathematics-grounding activities. The Asia-Pacific Education Researcher, 1-10.
  - Lin, F. L., Wang, T. Y., & Yang, K. L. (2018). Description and Evaluation of a Large-Scale Project to Facilitate Student Engagement in Learning Mathematics. Studies in Educational Evaluation, 58, 178-186.
  - Hsieh, C.J., Lin F.L., Sophie Chang C.H. (2021). Scaffolding the Concept of Area and Perimeter for Students With Severe Visual Impairment. Chinese Journal of Science Education, 29(4), 397-421. (In Chinese)

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- Teachers' learning:
  - Lin, F. L., & Hsu, H. Y. (2018). Using Mathematics-Pedagogy Tasks to Facilitate the Professional Growth of Pre-service Elementary Teachers. In Research Advances in the Mathematical Education of Pre-service Elementary Teachers, 3-17. Springer, Cham.
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- Educator-researchers' learning:
  - Yang, K. L., Hsu, H. Y., Lin, F. L., Chen, J. C., & Cheng, Y. H. (2015). Exploring the Educative Power of an Experienced Mathematics Teacher Educator-Researcher. ESM, 89(1), 19-39.
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Appendix MGA : R & D

Initial Phase

1. **Exemplary Game collection**

   e.g. Skemp rectangular number game

   e.g. Mathematics magics

2. **Provided by experienced teachers**

Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.

Appendix MGA : R & D

Development Phase

1. **Target topics :** Students' learning difficulties and important units
2. **Designing Workshop :** Designing Manual covers
   - Designing theories / principles and examples
   - Idea, key point and important units or necessary units
   - The structure of the contents in the design
   - Self-evaluation criteria

Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.

Appendix MGA : R & D

Development Phase

3. A module includes :

- The title of the activity
- The activity's mathematics concept, its related mathematical topic from the formal curriculum, and its corresponding status to the curricular materials
- The suitable subjects and grades for the activity
- The stages of preparation and the main activity.
- Worksheet

Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.

Appendix MGA : R & D

Development Phase

4. Candidates of designer

- Public: Name students' learning difficulties in mathematical topics and design lessons to help overcome these difficulties.
- Potential designers: experienced mathematics teachers, such as Center Counseling Team(CCT), Local Counseling Team(LCT)
- Academic invitation: gradual students and mathematics educators

5. Practical experiment : Trying out with students developmentally

Lin, F.-L., & Chang, Y.-P. (2019). Research and development of mathematics-grounding activity modules as a part of curriculum in Taiwan. In C. P. Vistro-Yu and T. L. Toh (eds.), School Mathematics Curricula, Mathematics Education-Asian Perspective (pp.151-168). Springer.

Thank you for listening.